ELEMENTARY STATISTICAL METHODS IN EDUCATION Updt

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ELEMENTARY STATISTICAL METHODS IN EDUCATION

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DEDICATION

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ACKNOWLEDGEMENT

We are sincerely indebted to Almighty Allah for endowing us with wisdom, knowledge, strength, and grace that enabled us to carry out this noble work successfully.

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Thanks also to our families and friends for their contribution. Thank you all.

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PREFACE

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In the ed

ucation sector, Statistics is referred to as "Educational Statistics" which means the application of the science of Statistics to solve educational problems. Therefore, various stakeholders in the education industry need reliable information in order to arrive at valid decision which is made possible through the use of Statistics. Thus, Statistics is used to actively perform specific educational tasks.

As a matter of fact, and specifically in the educational industries, educational statistics is used in various areas of endeavors such as to; determine the educational needs of the community in terms of school enrolment, withdrawal rate, and number of out-of-school rate. It is used to plan for valid estimates of human resources in terms of Teachers, Non-teaching Staff and Students.

Indeed, educational budgeting cannot be effective where accurate statistics is not available. As for Educators and Researchers, they would only be able to report research findings if they are competent and have good grip of educational statistics. This is because of the fact that, when reading research reports they would encounter Statistical symbols, concepts and ideas which must be read and understood. Hence, statistical knowledge and understanding are very necessary for all stakeholders especially those in the educational industries.

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Niger State College of Education, Minna

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FOREWORD

The desire for well-written books on Elementary Statistical Methods in Education cannot be overemphasized. The contents of this book were written by indigenous Authors who have addressed the auspicious lack of reference material on the subject matter. It is written with simple text, clear symbols, and well-explained terms. You will also find that this material was written with a younger audience in mind, meant to guide new researchers on how to collect and analyze data. Not only will professionals find the information useful, but many readers may realize they can easily apply statistical methods because they don't have to have deep knowledge of mathematics like other writers would require. You will come away from this text happier, convinced that statistical methods are not as challenging as you might think; it is also worth noting that suggestions for improvement will be appreciated.

Thank you.

Prof. Sirajo Abdulrrahaman (MNMS, MMAN, MNSMD)

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CHAPTER ONE

STATISTICA

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1.1

Introduction

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Almost all scientific fiel

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This chapter gives a brief meaning of Statistics, Educational statistics, Types of statistics and Purpose of statistics in education.

1.2Objectives

By the end of this chapter, you should be able to:

1. define statistics and educational statistics.
2. explain the types of statistics
3. describe clearly the purpose of educational statistics

1.3What is Statistics?

The term 'statistics' has different meaning to scholars. To some, statistics is a collection of numbers, charts, or data. To others, it is a branch of Mathematics, which deals with collecting, analysing and interpretation of data which will be utilized in valid estimates, predictions, and decision making. Similarly, Statistics is said to be a branch of applied mathematics that involves the collection, description, sorting, analysis, and drawing of conclusion from the qualitative or quantitative data obtained.

1.4Educational Statistics

Educational statistics simply means application of the science of statistics to solve educational problems. Educational Statistics is used in organizing, summarizing, presenting, and interpreting result and data from education measurements.

It is the organization and presentation of data, the measurement and description of individual or group performance, the design of experiments and testing of the significance of its results, as well as drawing of educational prediction.

The various educational stakeholders in education industries, like Educational Administrators, Teachers, Examination bodies, Researchers, Parents, and Students need information to perform their responsibilities, as such they need reliable information to arrive at valid decision through the use of statistics. So, Statistics is used to actively perform specific educational tasks.

1.5Types of Statistics in Modern Usage

Statistics has been classified into Descriptive Statistics and Inferential Statistics. The focus of descriptive statistics is on occurrences or the results of events that are described but no conclusions are drawn. In descriptive statistics, a variety of numerical, qualitative, or quantitative data are gathered, arranged, summarized, analysed, and presented. Mean, mode, median, standard deviation, range, percentage, and proportions are all dealt with in descriptive statistics.

Inferential statistics uses properties described in descriptive statistics to test hypotheses and draw conclusions. Statistical procedure, such as t-test, f-test and ANOVAs belong to these statistics.

As an education student, you need to study statistics, because of its benefits in making forecasting and taking a valid decision on educational matters or problems.

1.6Purpose of Educational Statistics

The various educational stakeholders like educational administrators, teachers, examination bodies, researchers, parents, and students need information to perform their responsibilities, such as the need for reliable information to arrive at valid decision through the use of Statistics. So, Statistics is used to actively perform specific educational tasks. As a student of education, you need to study statistics because of its benefits in making forecast and taking a valid decision on educational matters or problems. The following are some of the uses of statistics in education:

- Determining the educational needs of the community-population, age distribution, staffing existing schools etc.
- Planning for physical infrastructures, statistics issued in the number of classrooms required at a given period and preparing for future.
- Planning for Human resources: valid estimate should be made of Classes, Teachers, Students, and other non-teaching staff would be determined.
- Educational Budgeting: this entails keeping track of and calculating the total cost of human and material resource requirements. For smoother governance, every government requires accurate statistics.
- Comprehension and use of research: Every Educator and Researcher can only be able to report research findings, if only you are competent to grip statistical ideas and methods. Reading research reports means encountering statistical symbols, concepts and ideas which must be read and understand.
- Recordkeeping, test development, test scoring and reporting: All these require the knowledge and skills in collecting, organising, communicating analysis of data, and drawing inferences which are art of statistics. Statistical knowledge and understanding are very necessary for all stakeholders.

Students Activity

1. What is statistics?
2. Mention the two types of statistics.
3. Briefly explain four reasons for studying statistics in Education.
4. Clearly distinguish between descriptive statistics and inferential statistics.

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CHAPTER TWO

MEASUREMEN

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2.1Introducti

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easurement which affects the choice of statistical tools as well as interpretation and the conclusions, one may draw from the basic steps in educational research. The chapter will also discuss measurement, scale of measurement and steps involved in measurement process.

2.2Objectives

At the end of this chapter, you should be able to:

1. define measurement.
2. explain the types of measurement.
3. enumerate the steps involves in measurement process.
4. describe the use of rating scale in measuring behavior.

2.3Concept of Measurement

Measurement can be referred to as an act or systematic process of determining the size or amount of object or event by applying a defined rule or standard. Measurement always has two parts, Magnitude and Unit, Magnitude is the number while unit is the standard.

For example, if a length of table is 10cm, then 10 is the magnitude and centimeter (cm) is the Unit. Measurement is of two types; Physical measurement which requires the use of physical instrument such as rulers, tapes, stop clock, calendar, clinical thermometer, etc.

Psychological measurement sometimes referred to educational measurement which requires the use of educational tools such as classroom test, examination, observation etc.

2.4Process of Measurement

As Researchers in education, you are required to carry out some measurement processes.

These are steps involved in measurement processes: -

1. Identify a fact (phenomenon) of your study (e.g., teaching, learning, students, teachers, parents, society, school administers etc.).
2. Identify the aspect(s) or variable(s) of the fact (phenomenon) you intended to investigate (e.g., socioeconomic status of the parents, learning styles, teaching styles etc.).
3. Direct or indirect observation of the variables.
4. Appropriately select measurement scales.
5. Data collected are analyzed.
6. Interpret the data.

When number is assigned to represent the quantity of the variables observed direct or indirect with defined rules is known as scales of measurement or measurement scales.

2.5Measurement Scales

Scales of measurement is a defined sets of rules for assigning numerical scores to any directly or indirectly observed variable in order to know the quality or quantity of the variable. A measurement scale becomes valid if it has assignment of numbers to represent all things observed directly or indirectly.

There are four (4) levels of measurement (measurement scales)

2.5.1Nominal Scales

The lowest measurement scale is nominal scale, which classifies objects into two or more categories for identification needs. Numbers applied in this nominal scale has no magnitude or quantitative importance and cannot be used for any basic operations in mathematics (i.e., addition, subtraction, multiplication, and division). For example, the variable marital status used nominal scale; Not married = 0, Separated / Divorced = 1, Married = 2, Also if we assigned numbers to state of origin; Abia = 1, Bauchi = 2, Benue = 3 and Niger = 4, course code, MAT 224, EDU 402, EDME 408. All these are made for identification needs and cannot be added or subtracted or any other operations.

In nominal measurement, we do not play with size, order and other properties of the numbers assigned. They do not imply anything about the objects being measured or took no cognizance of the size, order and other properties of the numbers assigned. As a Researcher in education, it is good to know that two assumptions are made regarding the assignment of numbers in nominal scale. These are:

1. Different number is assigned for each object.
2. There exist a number assigned for observed or potential, quality, or quantity.

Therefore, the number values assigned in nominal scale stands for qualitative or quantitative differences in the part of the variables being measured.

2.5.2Ordinal Scales

This scale sometimes known as ranking scale, and it’s used for both classification and ranking of objects. The number values assigned aspect of the object being measured and put the object together in order from highest to lower, from most to least. In ordinal scale of measurement, we use the terms: grades position, merit awards, prize award etc. The only mathematical operation allows here or possible is correlation or association made regarding numbers assigned in the scale. These are:

1. Different number is assigned for each quality.

2. There exists a number for every observed or potential quality.
3. The number represent difference in some magnitude among the aspects of the variable being measured. Thus, the number values assigned in ranking scale shows the degree or difference of attribute possessed by the object.

2.5.3Interval Scales

This scale allows all mathematical operations to be carried out except division because the scale has no zero value. It is a scale that has the features of nominal and ordinal scale and as well as the quality of equal interval for the various grades.

For instance, if Ndagara, Liman, and Aliyu scored these marks, in a test; 30%, 40%, 50% respectively, there are equal difference between them. Furthermore, looking at five-point Likert Scale, a distance between the other points is equal.

1
2
4
5
3
Strongly Disagree (SD)
Undecided (U)
Strongly Agree (SA)
Agree
(A)
Disagree
(D)
1
2
4
5
3
Strongly Disagree (SD)
Undecided (U)
Strongly Agree (SA)
Agree
(A)
Disagree
(D)

This is an interval scale, there is a difference of 1 between two points, that is (SD) and (D) (3—2 = 1), (A) and (D) (4—3 = 1), (SA) and (A) (5—4 = 1). It involves assigning numbers to objects in such a way that you record equal differences in the objects of the attribute measured. Zero point is placed arbitrarily, which does not mean absence of the property measured.

The interval scale accommodates four assumptions regarding the assignment of numbers. They include:

1. Different number is assigned for each object.
2. There exists a number for every object observed or potential.
3. Numbers assigned indicate difference, in some magnitude among the aspects.
4. Numbers assigned represent equal units.

2.5.4Ratio Scales

Ratio scale is the highest and most precise measurement scale which is considered to be the most powerful among other scale of measurement because it has all the qualities of nominal, ordinal and interval scale. It has an absolute zero rather than an arbitrary origin. In other word, zero is true zero, indicating that, at that point the variable cease to exist. Addition, Subtraction, Multiplication and Division are possible in this scale. For example, we can say that weight of 8grams is twice as heavy as a weight of 4grams.

Regarding the assignment of numbers in this scale five assumption are made.

1. Different number is assigned for each object.
2. There exist a number for every observed or potential object.
3. The number values indicate differences in variable.
4. The number values assigned indicate equal units of counting.
5. Zero point at the beginning is true zero point.

The four measurement scales are very essential in the field of research work. To have a clear picture of the characteristics of measurement scale we can look into following table invented by Gilbert sax of university of Washington.

Scale	Definition	Understand Examples	Limitation
Nominal (least)	The scale that categorizes objects, persons, or events	Like names of places, objects, course code License plate numbers;	It cannot be used for specifying qualitative addition or difference among categories

Ordinal complex	The scale that ranks objects, persons, traits, or abilities instant recognition to equal of difference	Grade in letters (rating from excellent to fail), military rank, prize awards position and merit awards.	The Only Mathematical operation possible is corruption or comparison
Interval (most)	The Scale that has equal Differences between successive categories	The scale used variables such as, examination scores, temperature (centigrade and Fahrenheit) and calendar Date.	All Mathematical operation can be carryout in the scale except division and no zero value.
Ratio (complex)	The scale that represents highest and most precise measurement scale and has meaningful zero (0) value.	The scale is used in physical and behavioral sciences, Distance, height, and weight.	None except educational variables

2.6Rating Scales

You have learnt about measurement scales and based on your experience, To provide a suitable indication of where the observed behavior falls, rating scales for classifying behavior into categories or levels are a good idea. Since human behavior cannot be properly observed, educational researchers create rating systems that can be used.

Since values, attitudes, and beliefs are not physical objects like length, volume, mass, or weight, they are measured using rating scales. Because instruments are utilized, measurements of length, weight, height, and mass are accurate and objective. But because the outcomes are subjective and imprecise, gauging values, attitudes, and beliefs are not exact.

In education and social sciences, scales are extensively used; Likert-type, the Thurstone Scale, and the Guttmann Scale.

2.6.1The Likert-Type Rating Scale

The Likert-type rating scale, also known as the summated scale, was created by Rensis Likert and is named after him. This scale includes a collection of assertions or queries regarding the fact that is to be measured. Everyone is supposed to declare how much they agree or disagree with the statement or question. It is based on a five-point scale, however currently there exist Likert scales with less than five points. The replies are then added up or an average is calculated.

This will make it easier to assess how each person feels about the variable being assessed. For example, “I like Mathematics“.

4

3

2

1

4

3

2

1

A positive statement is scored with higher value while negative statement is scored with lower value, e.g.

"I like Mathematics"

1

2

23

14

1

2

23

14

Other terms are used in the place of agree and disagree as the situations may demand. For examples: Excellent, Very good, Good, Average, Poor, Extremely high, Usually true, Moderately, Slightly, Never true, Very important, Important, Less important, Not important.

2.6.2The Thurstone Scale

This rating scale, which is unidimensional, is used to monitor respondents' actions, attitudes, and feelings about a certain issue or topic. This scale includes of statements on a problem or subject, and each statement has a number value that reflects whether the respondent has a positive or unfavorable opinion of the subject. The researcher will give numerical values, and the explanation provided by these numerical values will indicate whether the respondents' reactions to the issue are positive or negative. The scale is frequently employed in the domains of sociology and psychology.

2.6.3The Guttmann Scale

Guttmann scale is sometimes called cumulative scaling or Scalogram analysis. It is one out of three uni-dimensional measurement scaled. A researcher make use of it to test how a person responds to a specific topic and measures how extremely positive or negative a person reacts topic or issue. Whether all the items measure every feature of the variable in issue or are facets of it is referred to as the variable's uni-dimensionality. For instance, if you are conducting research on attitudes toward mathematical literacy, you are aware that mathematics has many distinct facets. Trigonometry, geometry, algebra, abstract algebra, number theory, real analysis, etc. may be included. An individual could have a favorable attitude toward trigonometry but not geometry. Some people prefer actual analysis to abstract algebra. Two people could get the same score on this scale if they exhibit comparable patterns of interest on the same scale dimension as the issue. Alternatively, they may have been interested in the same variable's dimension. When the items on a scale are one-dimensional, we say that they constitute a perfect scale; however, when the intensity of the items on a perfect scale varies, we claim that the scale is completely repeatable. But when the response contains mistakes or contradictions, or when the scale contains mistakes.

It is said that the scale cannot be replicated. Having a perfect repeatable scale and estimating the extent are exceedingly tough tasks of reproducibility of a scale it is known as coefficient of reproducibility and given inform of an equation as:

Coefficient of reproducibility =
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If the calculated value is at 0.90 and above, we say the scale is reproducible. On other way round the scale is said not reproducible.

Student Activity

1. Explain the word measurement.
2. Mention two properties of measurement.
3. List four (4) levels of measurement.
4. Write a short on the following.
5. Nominal scales
6. Ratio scales
7. Ordinal scale.
8. Enumerate three (3) assumptions required in ratio scales.
9. List the steps involved in the measurement process.
10. Mention three (3) rating scales.

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CHAPTER THREE

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3.1Introduc

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description of numerical data.

Data collected in education can

be generated through many sourc

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frequencies, opinions, etc.

thus the process of re-organ

izing research data in such

way as to make them meaningful and useful is known as data organization.

In data organization, can be grouped in ascending order, descending order and frequency table for comprehension.

3.2Objectives

At the end of this chapter, readers should be able to:

1. arrange data in order of magnitude

2. arrange data in tabular form
3. arrange data in frequency distribution table

3.3Sequencing

The data arranged in order of magnitude is either in ascending or descending order.

Example 1:

Given that 10 students have the following scores in mathematics test: 5, 8, 13, 18, 12, 15, 6, 9, 7, 20

The ascending order gives: 5, 6, 7, 8, 9, 12, 13, 15, 18, 20

While the descending order gives: 20, 18, 15, 13, 12, 9, 8, 7, 6, 6, 5

Note that when data collected consist of names, they can be arranged in alphabetical order.

But when the data consists of objects, animals, event, etc., you may arrange based on its kinds or groups.

3.3.1Rank Order

If the data obtained shows rank order, it sometimes may be described with descriptive statistics such as histograms, pie chart, frequency polygon, and bar charts. To illustrate this order, considering the scores on an intelligence mathematics test for six students.

Table 1. Rank Order in Test Scores

Students	Score	Rank
1	153	1
2	142	2
3	140	3
4	127	4
5	118	5
6	109	6

The scores can be rank order from the most intelligent to the list intelligent as been carried out in *Table 1.* In other way, it could be ranked from least scores to highest intelligent, assignably 109 to be rank of 1, the intelligent test score of 118 be assigned as rank of 2, and so on.

3.4Tables

The data collected can be organized or arranged in a Table either by categories or frequencies. An example of a two-dimensional representation of statistical data is a table. Table must have general title, column title and row title, with a source note on it bottom.

For example, academic planning unit C. O.E Minna has on their records numbers of students admitted from 2010 to 2016, and distribution of Lecturers in each department as at year 2022.

Example 1

Table 2: Students' Admission in Niger State College of Education, Minna, 2010–2017

S/N	Year	Males	Females	Total
1	2010	1200	1050	2250
2	2011	1557	1400	2957
3	2012	1300	1150	2450
4	2013	1600	1400	3000
5	2014	1504	1335	2839
6	2015	1706	1540	3246
7	2016	1245	1150	2395

Source: Academic Planning & Statistics Units.

Example 2

Table 3: Distribution of Lecturers in School of Sciences

S/N	Department	No. of Lecturers
1	Biology	15
2	Chemistry	10
3	Computer Science	06
4	Integrated Science	11
5	Mathematics	12
6	Physical and Health Education	18
7	Physics	07
	Total	79

Source: Dean's office.

3.5Frequency Distribution Table

Basically, there are two kinds of frequency distribution tables; ungrouped and grouped frequency distribution tables. These tables display the frequency with which each value, item, or score appears in a certain distribution. The frequency table is used as a means of summarizing and highlighting important aspect of mass data in a more meaningful and interpretable form. It consists of three columns, score/items, tally mark and frequency.

3.5.1 Ungrouped Frequency Distribution Table

In organizing and presenting data of ungrouped data by a Researcher, there are some procedures to be followed. These procedures are:

*Step 1:*Arrange the items or scores in ascending or descending order.

*Step 2:*In column 2, tally each item or stroke for each time you come across the item or score in the array. The fifth time cross the four strokes already put down.

*Step 3:*Write number of strokes for each item or score in third column. To illustrate the organization and presentation of ungrouped frequency distribution table, let us look into the following examples.

Example 3.

The followings are test scores of students in 2nd semester Examination

20, 30, 10, 20, 20, 10, 30, 30, 30, 10, 60, 70, 40, 30, 30, 80, 60, 50, 90, 60.

Table 4: Test Scores of Students Examination

Score X	Tally Marks	Frequency
10	III	3
20	III	3
30	IIII I	6
40	I	1
50	I	1
60	III	3
70	I	1
80	I	1
90	I	1
	Total	**20**

Example 4.

In a study of opinion of Pre-service Mathematics Teachers' Efficacy of Problem-solving Attitude on Achievement in Mathematics in Niger State College of Education, Minna. The following score of 30pre-service teachers were obtained for two items in Problem-Solving Attitude Questionnaire (PSAQ). The questionnaire developed is on five-point scale: Strongly Agree (SA) = 5, Agree (A) = 4, Disagree (D) = 3, Strongly Disagree (SD) = 2, Undecided (U) = 1.

Table 4.1. Scores obtained by 30 Pre-Service Teachers on the Two Items

S/N	Item	SA	A	D	SD	U
		5	4	3	2	1
1.	Most people think that I am objective and logical	7	10	8	2	3

2.	I really enjoy solving new problems	12	6	4	6	2

As a Researcher, you need to know the number of responses in each item. To carry out this, you need to prepare an ungrouped frequency distribution table and can be presented in Table 4.2 and 4.3

Table 4.2: Ungrouped Frequency Distribution Table for 30 pre-service teachers for item I: "Most people think that I am objectives and logical"

Table 4.2: 30 Pre-Service Teachers on Item One

Scores	Tally	Frequency
5	IIII II	7
4	IIII IIII	10
3	IIII III	8
2	II	2
1	III	3
Total		**30**

Table 4.3. Ungrouped Frequency Distribution Table for 30 Pre-service Teachers on Item 2: "I really enjoy solving new problems", can be shown as below:

Scores	Tally	Frequency
5	IIII IIII II	12
4	IIII I	6
3	IIII	4
2	IIII I	6
1	II	2
Total		**30**

3.5.2 Grouped Frequency Distribution Table

A Researcher may obtain large number of scores and may be very difficult to list all of them. For easy and faster approach is good to group the scores. To prepare a grouped frequency distribution table, you need to adopt the following steps:

Step 1: Grouping the scores into class intervals.

Before the grouping, it is worthy to understand some basic concepts involved. These are class interval, class size, class limits and class boundaries.

A class interval is a small group of score within uniform scores in each group. Two extreme scores are involved, the lowest scores and highest score. While the number of scores within a class interval is known as the class size. The extreme scores in a given class interval is known as a *class limit.* The highest score is *upper class limit,* and lowest score is *lower class limit.* For example, a class interval 3–6, the class size is 4. The lower-class limit is 3 and 6, is the upper-class limit.

Also, *class mark (*class mid-point) is obtained by adding the lower and upper-class limits, divided by two. Thus, the class mark of the class interval 3—6 is $\frac{3+6}{2}=4.5$

Class boundaries is obtained by adding 0.5 to the upper-class limits and subtracting 0.5 from lower class limit. From the class interval 3 - 6, the lower-class boundary is 2.5 and upper-class boundary is 6.5. Below shows some class limits and their corresponding class boundaries.

Table 4.4:

Class Limit	Class Boundaries
3—6	2.5—6.5
7—10	6.5—10.5
11—14	10.5—14.5
15—18	14.5—18.5
19—22	18.5—22.5

The Researcher must determine the class size to be used before grouping the score. To carry out that you require the following procedures:

Step 1: The range of the set of scores between the highest score and lowest score is determined.

Step 2: Range is divided by 10, you usually have between 10 to 15 classes of scores (groups).

Step 3: The result obtained is approximated to the nearest odd number.

Step 4: Tally the score in column 2.

Step 5: Put down the frequency occurrence of each class interval in column 3.

Let us illustrate with the data obtained by a researcher after administered post-test in a study.

45 22 33 51 34 22 66
40 37 42 46 29 41 33
38 47 43 39 57 65 38
44 31 11 38 45 32 41
57 59 46 25 54 39 40
42 62 35 17 43 55 20
53 57 37 43 32 27 45

Step 1: Group the scores into class intervals. To do this, you need to determine the class size. In determining the class size, you should ensure that the number of class intervals are not fewer than 10 or more than 20. Class size should be at 2, 3, 5, _ _ _ 10 and multiples of 10 are preferred.

From the data above, lowest score is 11. Let arrange 6 classes of 10 class size.

The grouped frequency distribution table for the data above is presented in Table 5.

Table 5: Data for Post-test Score in Examination

Class Interval	Tally	Frequency
10—19	II	2
20—29	IIII I	6
30—39	IIII IIII IIII	14
40—49	IIII IIII IIII II	17
50—59	IIII III	8
60—70	III	3
Total		**50**

Student Activity

1. Why are statistical data put into tables?
2. How many types of tabulation are there? Name them.
3. What is frequency distribution tables?
4. How many kinds of frequency distribution tables are there? Name them.
5. What are the characteristics of general statistical table?
6. Explain the following concepts:
7. Class interval,
8. Class mark
9. Upper class limits
10. Lower class limit.
11. The following table lists some students' MAT 203 exam results. Create a frequency distribution table with the scores.

20, 25, 28, 22, 24, 25, 30, 25, 26, 21, 22, 24, 29, 30, 27, 28, 25, 23, 21, 22, 29, 23, 20, 24, 27, 29, 26, 25, 25, 25.

1. A Researcher conducted a study and administered post-test. The scores are presented below, organize the results into a frequency table.

55, 39, 48, 60, 46, 52, 51, 33, 58, 65, 55, 62, 59, 53, 50, 55, 52, 48, 58, 65, 60, 55, 60, 51, 35 36 55, 54, 68, 68, 55, 45, 55, 52, 62, 56, 59, 47, 39, 46, 60, 58, 65, 33, 42, 53, 40, 47, 34, 61, 42, 48, 38, 48, 50.

References

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10

CHAPTER FOUR

GRAPH

ICAL REPRESENT

ATION OF DATA

4.1Introd

uc

tion

In the

previous cha

pter,

you hav

e learnt how to collect

and tabulate statistical data.

This chapter, will look into h

ow to present frequency distribu

tion data into graph for easy s

eeing, convey information and

also assists in the understa

nding of data.
All graphs m

ust be properly scaled in o

rder to convey correct information. Graphical methods to be discussed include Pie chart, Bar chart, Histogram, and Frequency polygon.

4.2Objectives

By the end of this section, you should be able to:

1. Construct Pie chart,
2. Draw bar chart, histogram, and frequency polygon
3. Draw a cumulative curve
4. Draw a table of relative frequency distribution (percentage distribution)
5. Construct a relative frequency histogram

4.3Pie Chart

Pie chart is sometimes known as circle graph. It is a chart that arrange, events, objects or items in a circle form end used for both discrete and continuous data. Each occurrence or item is represented by a sector of the circle, with the angle occupied at the sector's center corresponding to the frequency of the occurrences.

Example 1:

A Researcher who is interested in studying the performance of students in various department, can present the data using pie chart. For example, the examination Officer of School of Sciences presents the average performance of Students in various Departments in the School as follows: Biology = 55, Chemistry = 30, Computer Science = 40, Mathematics = 25, Integrated Science = 60, Physics = 20.

Solution

The Researcher following the procedures is to present the data in pie chart.

Step 1: Sum up all performance in the six departments, thus:

55 + 30 + 40 + 25 + 60 + 20 = 230.

Step 2: calculate the angle that is subtended at the center by each item.

Biology $\frac{55}{230}\times 360^{\circ}= 86.9^{\circ}$

Chemistry $\frac{30}{230} \times 360^{\circ} = 46.96^{\circ}$

Computer Science $\frac{40}{230} \times 360^{\circ} = 62.61^{\circ}$

Mathematics $\frac{25}{230} \times 360^{\circ} = 39.13^{\circ}$

Integrated science $\frac{60}{230} \times 360^{\circ} = 93.91^{\circ}$

Physics $\frac{20}{230} \times 360^{\circ} = 31.30^{\circ}$

Step 3: Use a pair of compasses, draw a circle with any convenient radius and indicate the center of the circle.

Step 4: Mark the angles corresponding to each item using your protractor.

Step 5: Label the six sectors obtained, that is Biology, Chemistry, Computer Science, Mathematics, Integrated Science and Physics.

Fig. 4.1: A pie chart of average performance of departments

Example 2:

The data shows the numbers of lecturers in each department at certain College of Education

Department	Number of Lecturer
Arabic	14
English	12
French	8
Hausa	4
Nupe	2
Total	**40**

Solution

To construct the pie chart as earlier outline some steps to be followed. Firstly, sum-up number of lecturers, which gives 40. This numbering 40 lecturers correspond to the number of degrees in circular are 360°

i.e.,

Arabic: $\frac{14}{40}\times 360=126^{\circ}$

English: $\frac{12}{40}\times 360=108^{\circ}$

French: $\frac{8}{40}\times 360=72^{\circ}$

Hausa: $\frac{4}{40}\times 360=36^{\circ}$

Nupe: $\frac{2}{40}\times 360=18^{\circ}$

Now use your protractor to obtain required angles for each item after drawing a circle of any continent radius.

Figure 4.7

4.4Bar Chart

Bar chart is known as bar diagram or bar graph, and which contains bar that stand out exclusively from one another. This shows that the measurement scales are not continuous but discrete data. Bar diagrams convey the frequency of cases in each group relative to each other. It has two axes vertical and horizontal base. The vertical axis is known as the ordinate and horizontal axis is called abscissa. The bars can be vertical or horizontal and the line bars or columns are of equal width, but the height varies according to the proportion of the data.

The data in last example can be represented in a bar chart by the following steps:

Step 1: Choose a convenient scale to draw the two axes (vertical and horizontal).

Step 2: Make out the height of each section based on the chosen scale.

Step 3: Draw out the bars of each to represent the height.

Example: Draw a bar chart for the information given in

Table 4.3. The table is provided below

Department	Number of Lecturer
Arabic	14
English	12
French	8
Hausa	4
Nupe	2
Total	40

00Solution

Figure 4.8

A Simple bar chart consist of various components, which each component represents a section and each section corresponds in size to the magnitude of the item it stands for.

4.5Histogram

It is created by graphing the frequencies of the respective class interval against the class borders. It is mostly used to represent continuous data.

The following procedures should be followed in order to create a histogram.

*Step 1:*create a frequency distribution table with the class interval, frequencies, and class borders changed.

*Step 2:*Select appropriate scales for both axes and draw the vertical and horizontal axes

*Step 3:*is to label the axis using the selected scales.

*Step 4:*On each border, draw rectangular bars with heights according to the frequencies.

*Step 5:*Draw arrows to indicate what is on the vertical and horizontal axis.

To draw the histogram, you then adopt the listed above procedures 1—5.

To illustrate the histogram scores of 80 students in MAT 224 test at the end of 2^{nd} semester exams in a certain College of Education as follows:

Scores	Number of Students
50—52	5
53—55	11
56—58	14
59—61	10
62—64	8
65—67	7
68—70	6
71—73	9

77—79	5

Table 4.1: MAT224 Test Score for 2nd Semester Exams

S/N	Class Interval	Class Boundary	Frequency
1.	50—52	49.5—51.5	5
2.	53—55	51.5—55.5	11
3.	56—58	55.5—58.5	14
4.	59—61	58.5—61.5	10
5.	62—64	61.5—64.5	8
6.	65—67	64.5—67.5	7
7.	68—70	67.5—70.5	6
8.	71—73	70.5—73.5	9
9.	74—76	73.5—76.5	5
10.	77—79	76.5—79.5	5
	TOTAL		**80**

Fig. 4.3

Example:

The following table indicates the distribution of mark scored by a class of 100 students

Table 4.4: Mark Scores by Students

Marks	Number of students
11—15	20
16—20	11
21—25	12
26—30	28
31—35	16
36—40	13

Example: Draw a histogram of the distribution

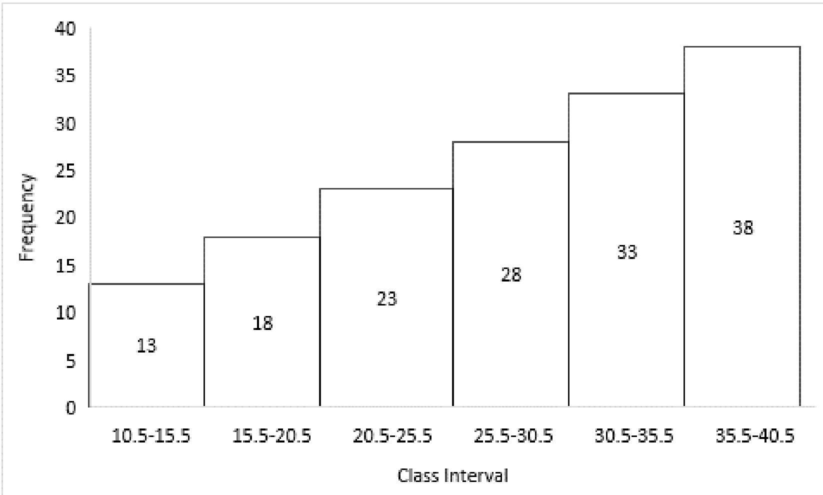
Solution

A new table is prepared showing the class boundaries and class mark

Table 4.5: Adjusted Data of Mark Scores by Students

Marks	Class boundary	Class mark	Frequency
11—15	10.5—15.5	13	20
16—20	15.5—20.5	18	11
21—25	20.5—25.5	23	12
26—30	25.5—30.5	28	28

31—35	30.5—35.5	33	16
36—40	35.5—40.5	38	13
Total			100



4.6Frequency Polygon

This is a type of graph of a frequency distribution which is obtained by plotting the class frequencies against the class marks. It is polygon because the mid-point of the tops of the rectangles in the histogram are connected.

A frequency polygon is constructed by following the procedures below:

*Step 1:*Draw both axes (i.e., vertical, and horizontal).

*Step 2:*Mark out the frequencies along the vertical axis and the mid-points of class intervals on the horizontal axis.

*Step 3:*Plot the frequency of each class interval at the appropriates height as a point above the mid-point of interval.

*Step 4:*Join these points with straight lines.

*Step 5:*Connect the first and last dots with the horizontal axis at the mid-point before the first dot and the one after the last dot.

Using the data in Table 4.1 to present a frequency polygon by adopted step 1—5.

Fig. 4.4: frequency polygon of data in table 4.1

4.7Frequency Curve

Frequency curve is sometimes called *smoothed frequency polygon.* It is smooth curve that joins the middle of the tops of the histogram. It is similar to the frequency polygon, the only difference is that frequency curve is a smooth curve while, the frequency polygon is a line segment. In drawing the frequency curve is the midpoint of the class intervals against the cumulative frequency. To illustrate this graph using the data in *Table 4.2.*

Table 4.2: shows the scores in Achievement Test

A	B	C	D	E
Interval	Exact limit	Frequency	Midpoints	Cumulative frequency
2—4	1.5–4.5	2	3	2
5—7	4.5–7.5	8	6	10
8—10	7.5–10.5	7	9	17
14—16	13.5–16.5	17	15	34
17—19	16.5–19.5	7	18	41
20—22	19.5–22.5	7	21	48

Fig 4.5: Frequency Curve

4.8Relative Frequency Distribution

The relative frequency is how often a specific type of event occurs within the total numbers of observation. Relative frequency does not make use of raw counts, rather they relate the count for an event to the total number of events using percentage, proportion, or fraction. It is the actual frequency of the class divided by the total frequency of all classes.

4.8.1Computation of Relative Frequency

To calculate for relative frequency, you must know the followings:

- The count of events, for a particular category
- The total number of observations (events)

The formular is given as:

Relative Frequency (RF) $=\frac{Countofevent}{\begin{array}{c} Totalnumberofevent\\ \end{array}}$

Let us use the Table 4.2 data to calculate the relative frequency.

S/N	Class interval	Frequency	Relative	Percentage

i.	2—4	2	\\frac{2}{98}=0.02	2
ii.	5—7	8	\\frac{8}{98}=0.08	8
iii.	8—10	7	\\frac{7}{98}=0.07	7
iv.	14—16	17	\\frac{17}{98}=0.17	17
v.	17—19	7	\\frac{7}{98}=0.07	7
vi.	20—20	7	\\frac{7}{98}=0.07	7

When the frequencies in Table 4.2 are replaced by corresponding relative frequency, the result obtained is known as relative frequency distribution.

The relative frequency distribution graph can be draw from the histogram or polygon by using the x-axis for class interval and y-axis for relative frequency.

For example, use the data below to construct

1. a relative frequency percentage distribution
2. a relative frequency histogram
3. a relative frequency polygon.

Solution

Scores of 100 Female Students in Department of Mathematics

Scores	Frequency	Relative frequency	Percentage
X	F	RF	10
50—52	6	0.05	5
53—55	18	0.18	18
56—58	42	0.42	42
59—61	27	0.27	27
62—64	8	0.08	8
TOTAL	100	1	100%

Note that each class frequency is calculated in relation to the total frequency of all classes, which is 1.

FIGURE 4.6: SCORES

Students Activity

1. Newly admitted Mathematics Students of ABU Zaria spent a total of N60,000.00 with the following details:
2. Tuition fees = N10,000.00
3. Game fees = 5,000.00
4. Clinic fee = N2,000.00

5. Course materials = N15,000.00
6. Accommodation = N10,000.00
7. Stationaries = N5,000.00
8. Feeding = N10,000.00
9. Notebooks = N3,000.00

Construct these exercises in pie chart.

1. Consider the following scores obtained by a Researcher by administering 40 Students in post-test.

30 25 54 50 12 5 18 40
21 25 55 13 40 3 46 3
23 21 34 49 18 8 48 18
39 27 37 30 15 5 23 46
21 21 33 35 5 8 45 38

1. Assuming a class size of 5, create a frequency distribution table for the data.
2. Draw a bar chart.
3. Draw a Histogram.
4. Draw a frequency polygon.
5. Explain a relative frequency
6. Draw the relative frequency histogram for the data below.

Masses of 100 Students at a certain Department of Mathematics

Mass (x) Kg	Number of Students F
30—32	8
33—35	42
36—38	27
39—41	5
42—44	18

1. Compute the relative frequency and relative percentage of the data given in Table.

Table 4.1: Scores Obtained in Statistics by 90 Students

Mark X	Frequency F
51—55	2
56—60	12
61—65	11
66—70	13
71—75	22
76—80	7
81—85	10
86—85	5
91—95	5
	90

References

Awotunde, P. O. & Ugodulunwa (2002). An Introduction to Statistical Methods in Education. Printed and published in Nigeria by Fab Anieh (Nig) Ltd.

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CHAPTER FIVE

SIM

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5.1Introduc

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This assists in minimizing erro

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th simple percentage. Simple

percentage is another form

of descriptive statistics u

sed in analyzing data.

5.2Objectives

At the end of this chapter, you should be able:

1. round off data
2. convert data to percentage.

3. Calculate actual error
4. Compute percentage error

5.3Rounding off Data

Statistical data collected through measurement or counting are approximated into a desired degree of accuracy, either rounding off upwards or downward. Rounding off is the method of approximating a number to the nearest unit, hundred, thousand, million, decimal places or significant figures.

The digits 1, 2, 3, 4 are rounded down and they are called zero. While the digits 5, 6, 7, 8, 9, are rounded up called 1 and added to the next digit.

Let us illustrate how to round off numbers to the nearest tens, hundreds, or thousands.

Examples

1. Approximate each of the following to the nearest tens, hundreds, and thousands.
2.
3. 13657
4. 76523
5. 13,657 becomes 13660 the nearest ten, round up
6. 13657 becomes 13700 the nearest hundred round up
7. 13657 becomes 14,000 to the nearest thousands round up.

1. 76523 = 76520 to nearest ten round down
2. 76523 = 76500 to nearest hundred round down
3. 76523 = 77000 to nearest thousand round up.

Note that when measurement is made, the approximation or round off makes the data collected clear and more understandable. It also assists, in minimizing cumulative errors when a large number of operations are carried out.

5.4Decimal Point and Significant Figure

Through statistical manipulation, the measurement-based statistical data may be made more understandable or acceptable. For example, they can be rounded to a predetermined number of significant figures or decimal places.

5.4.1Significant Figures

The significant figure, abbreviated as (sf), is calculated starting from the non-zero numeral located to the number's left. The leftover numbers are removed using the following guidelines once the necessary number of significant figures has been recorded:

If a set of numbers is to be eliminated and the first digit is a 5 or above, the last digit is raised by 1.

For example:

1. Approximate 7.4543 to two, three, significant figures

7.4543 = 7.5 to 2 sf

7.4543 = 7.45 to 3 sf

2. Approximate 0.00758 to one sf, two sf.

1. 0.00758 the zero before the real number is not a significant figure. The rounding off will take place after the first real number.

0.00758 = 008 to 1 sf

1. 0.00758 = 0.0076 to 2 sf.

Note that the number zero is only significant if only situated after any non-zero real number in the whole number part e.g., 3406, the zero here is significant, but in 0.069, 15.40 and 0.000056 are zeros that are not significant.

5.4.2Decimal Point

Decimal point is sometimes referred to decimal place and abbreviated to *dp.* These are counted to the Right of the decimal point and contained the same rules of rounding off in significant figures. For example,

Round off each of the numbers to

1. one decimal place
2. two decimal places
3. 0.005
4. 6.5020
5. 0.005 = 0.0 to 1 dp

= 0.01 to 2 dp

1. 6.5020 = 6.5 to 1 dp

   = 6.50 to 2 dp

5.5Percentage

Percentage is a useful statistic used for describing information obtained. It is good in describing the features of two or more groups of people or objects where the number of people or objects in the groups are different. At this particular period, reporting only the frequency of such features is not enough. But if the frequency of people in each group who possess the relevant features is converted to the percentage, then the group can be compared. Because the percentage treats the groups as though they are of the same size (i.e., 100).

Simple percentage is given as = $\frac{\mathrm{N}\mathrm{u}\mathrm{m}\mathrm{b}\mathrm{e}\mathrm{r}\mathrm{o}\ldots}{\mathrm{T}\mathrm{o}\mathrm{t}\mathrm{a}\mathrm{l}\mathrm{n}\mathrm{u}\mathrm{m}\ldots}$ 100

A Researcher obtained data in School of Arts and Social Sciences and is interested in comparing the performance of students in the different department in the School as shown in Table 5.1.

Table 5.1: Shows 2020/2021 session number of Students admitted and those who passed without any carry over.

Department	No. of Admitted	No. Passed
Economics	850	400
Geography	250	200
History	180	150
Social Studies	900	300

The percentage of Students who passed without any carry over in each department of School of Arts and Sciences is as follows:

1. Percentage who passed Economics $=\frac{400}{850}\times \frac{100}{1}\times =47.05^{\circ}\%$
2. Percentage who passed Geography $=\frac{200}{250}\times 100=80^{\circ}\%$
3. Percentage who passed History $=\frac{150}{180}\times 100=83.33^{\circ}\%$
4. Percentage who passed S/studies $=\frac{300}{900}\times 100=33.33^{\circ}\%$

If you compare the number of frequency of Students who passed in Economics department without any carry over, which is (400) and those in the Geography department that is you may be tempted to conclude that performance in Economics is better than Geography. But if you use the percentage values you may see that performance in Geography is better than performance in Economics.

5.5.1Percent Error

Percent Error is degree or level of difference between estimate value and the actual value in comparing to the actual value and is expressed as a researcher on taking decision on the acceptable level of error. The error may be either positive or negative.

Let us look into this example. A student in a lab measure the lenght of alminium glass and accidentally records 10 M if the actual lenght is 12 M

To calculate the percentage error, the followings steps shall be adopted.

1. Subtract the actual value from the estimate value
2. Divide the result obtained from step I
3. Multiply the result by 100 to obtain total percentage.
4. Add percent or % symbol to report your result (i.e. percent error values)

To solve for this example, you use the formula

Percentage error (P.E) =\\frac{\\mathrm{E}\\mathrm{s}\\mathrm{t}\\mathrm{i}\\mathrm{m}\\mathrm{a}\\mathrm{t}\\
\\mathrm{A}\\mathrm{c}\\mathrm{t}\\mathrm{u}\\mathrm{a}\\mathrm{l}\\mathrm{V}\\mathrm
(\\mathrm{A}\\mathrm{V})}
{\\mathrm{A}\\mathrm{c}\\mathrm{t}\\mathrm{u}\\mathrm{a}\\mathrm{l}\\mathrm{V}\\mathr
100

\\boldsymbol{S}\\boldsymbol{o}\\boldsymbol{l}\\boldsymbol{u}\\boldsymbol{t}\\boldsymbol{i}\\boldsymbol{o}

Actual value = 12 M

Estimated Value = 10 M

*Step I:*10\\mathrm{M}\\hbox{--}12\\mathrm{M}=\\hbox{--}2\\mathrm{M}

Step II: Divide the result with actual values - \\frac{\\hbox{--}2\\mathrm{M}}{12\\mathrm{M}}=\\hbox{--}0.17M

Step III: Multiply the result by 100 to obtain total percentage 0.17 \\times 100 = 17

Step IV: Add percent –17%.

\\boldsymbol{E}\\boldsymbol{x}\\boldsymbol{a}\\boldsymbol{m}\\boldsymbol{p}\\boldsymbol{l}\\boldsymbol{

A table of lenght 25cm was measured by a male student to be 24.6cm. Find the percentage error.

The actual error is 25—24.6 = 0.4cm

Percentage error = \\frac{0.4}{25}\\times 100=0.016\\times 100

=1.6\\mathrm{\\% }

\\boldsymbol{E}\\boldsymbol{x}\\boldsymbol{a}\\boldsymbol{m}\\boldsymbol{p}\\boldsymbol{l}\\boldsymbol{

A block which weighs 30.5kg is obtained to have weighed 32.6kg. Find the percentage error.

\\boldsymbol{S}\\boldsymbol{o}\\boldsymbol{l}\\boldsymbol{u}\\boldsymbol{t}\\boldsymbol{i}\\boldsymbol{o}\\

The actual error = 32.6—30.5 = 2.1kg

The percentage error = =\\frac{2.1}{30.5}\\times 100=6.9\\%

Student Activity

1. What is rounding off data?
2. What is benefit of rounding off data?
3. Round off each of the following numbers into nearest unit, ten, hundred and thousand.
4. 24567
5. 88732

1. Round off the following numbers into
2. 1 dp
3. 3 dp
4. 1 sf
5. 3 sf
6. 00.7568 *b.* 10.768 *c.* 0.0759
7. The number of students in different levels who are intelligent and the total number of students in such levels are given bellow:

	TOTAL NUMBER OF STUDENTS	NUMBER OF INTELLIGENT STUDENTS
100LEVEL	30	6
200LEVEL	25	2
300LEVEL	20	4

What percentage of students that are gifted in each level?

1. In research to compare academic withdraw rates in department of chemistry in three levels. A researcher obtained the following data.

	Numbers of Admitted	Number of Withdrawn
300LEVEL	80	40
200LEVEL	30	10
100LEVEL	50	20

1. What is the percentage withdrawing rate in each of the three levels?
2. What is the percentage of the total number of the student in the three-level withdrawn?
3. A researcher was to subtract 10 from a certain number but mistakenly added 15 and obtain the result 140. Determine the percentage error during the data collection.
4. The student in Mathematics laboratory measured the length of electrical wire as 16.55, while the length is 16.25. Calculate the percentage error and correct the result to 1 decimal place.
5. If the age of my wife is 45 years is recorded as 61 years, calculate the percentage error to 2 significant figures.

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Razaq D. & Ajayi, O.O (ND) Research Methods and Statistical Analysis.

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CHAPTER SIX

MEASURES OF CENTRA

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6.

1Introduction

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s sometimes referred to as meas

urements of location or central

value. It entails identifying a

single measure that characteri

zes a collection of scores. Th

ese metrics are used by the

researcher to categorize a g

roup of data that each have

a single value or number. It comes in handy a lot when describing overall performance in the educational industry as a researcher. The outcome produced by the computation of a locational measure captures or stands for the usual or average score acquired by a set of participants. That is, a whole collection of scores may be represented by a measure value.

The most popular descriptive statistics of this kind, mean, median, and mode, will be covered in this chapter. Each measure of central tendency is a rough estimate for a distinct scale.

6.2 Objectives

By the end of this chapter, you should be able to:

1. describe the three most commonly used measures
2. compute the three most commonly measures of location.
3. explain major characteristics of mean, median, and mode and their uses in research studies.
4. Compute the mean using the assumed mean
5. Calculate the mean using the coding factor
6. Compute mean using coding factor

6.3 Mean

The mean, which is sometimes referred to as the arithmetic average, is derived by summing the scores for each topic individually and dividing the result by the number of subjects. Typically, the sign for mean is used x̄ (read x 'bar'). The formula is given as:

Mean = $\bar{x} = \frac{\sum \mathbf{X}}{\mathbf{n}}$

Where $\bar{x}$ = the arithmetic mean

Σ = Greek letter sigma meaning "sum of"

X = scores

n = number of sample size. This is for ungrouped data.

For the grouped data, the formula for calculating the mean is given as:

$\bar{x} = \frac{\sum \mathbf{f}\mathbf{X}}{\sum \mathbf{f}}$

Where fx = the products of frequencies and individual scores

Σf = the summation of frequencies.

The Mean is sub-divided into: Assumed Mean, Geomettric Mean, Quadratic Mean and Harmonic Mean.

- *Assumed Mean (AM):* this method is carryout by deducting each value of a variable from an assumed mean which could be the class mark of any group or class interval. Assumed mean can be employed for both ungrouped and group Data.

$\overline{\boldsymbol{x}}=\mathrm{A}+\frac{\sum fd}{N}$

Where A is the assumed mean

d is the deviation from assumed mean (A) for each of the value of variable

ie $\overline{\boldsymbol{x}}-\mathbf{A}$

\\boldsymbol{N} is the sample size \\sum f

- *Geometric Mean (GM):* The geometric mean of n positive value is the root of their product for a set of observations:

x_{1}, x_{2,}x_{3},----,x_{u},geometric mean is calculated as:

\\mathbf{G}\\mathbf{M}=\\sqrt[]{\\left(x_{1}\\right)\\left(x_{2}\\right)\\left(x_{3}\\right)\\left(x_{u}\\right)}

Quadratic Mean (QM): it is the square root of the mean of the sqaures of the given values, it is calculated as:

\\mathbf{Q}\\mathbf{M}= \\sqrt[]{\\frac{{\\mathrm{x}_{1}}^{2}+{\\mathrm{x}_{2}}^{2}+{\\mathrm{x}_{3}}^{2}+---,{\\mathrm{x}_{\\mathrm{n}}}^{2}}{\\mathrm{N}}}

- *Harmonic Mean:* it is the recent reciprocal of the arithmetic mean of the rciprocal of the observation and it is calculated as:

\\mathbf{H}\\mathbf{M}=\\frac{\\mathrm{N}}{\\Sigma _{i}n=1}=\\frac{N}{\\frac{1}{x_{1}}+\\frac{1}{x_{2}}+\\frac{1}{x_{3}}+---+\\frac{1}{x_{n}}}

Major Properties of Mean

1. It is the arithmetic mean of the measurement in the data set.
2. It is only one value for a data set.
3. The extreme measurement influenced its value and trimming can assist to decrease the degree of outlier influence.
4. In determining the mean of the complete data set, means of subjects can be combined.
5. It is only used for quantitative data.

6.3.1Mean of Ungrouped Data

Considering the following scores in Geometry Achievement Test

12, 0, 13, 6, 2, 3, 9, 8, 14, 6

To calculate the mean, the simple formula $\bar{x}$=\\frac{\\sum \\mathbf{X}}{\\mathbf{n}} can be used when the number of scores is small.

This is shown below:

Mean = $\bar{x}$ = \\frac{12+0+13+6+2+3+9+8+16+6}{10} = 7.5

In this case, the mean value of students which stand at 7.5 typically represent the overall performance of the group. This simple formula can only be applied when the scores are fewer and not arranged in a frequency table.

However, if the scores are arranged in a frequency table, the second formula provided can be used that is $\bar{x} = \frac{\sum \mathbf{f}\mathbf{x}}{\sum \mathbf{f}}$. To apply this formula, you have the following steps to be followed:

*Step 1:*Create a frequency distribution table as indicated in table 6.1. That is first column for the scores while the second column for the frequencies.

*Step 2:*Develop a third column for the product of each score with the corresponding frequency provided as (*Fx)*

*Step 3:*Sum-up the frequencies of the scores in column 2, (that is *Σf)*

*Step 4:*Add-up the product of each score with its frequency, that is *Σfx*

*Step 5:*Substitute in the formular that the results obtained in step 3 and step 4

Thus, the procedures can be illustrated using the example below.

Suppose the scores below are obtained for 30 students in geometry performance test

5, 8, 10, 8, 1, 7, 7, 10, 8, 8, 7, 10, 7, 7, 7, 5, 4, 3, 2, 4, 3,2, 4, 7, 3, 9, 2, 6, 5, 6, 5, 9, 6, 9, 6, 6, 9, 5, 3, 2, 2.

Table 6.1: Scores in Geometry Performance Test

Column 1: Score X	Column 2: Frequency	Column 3: FX
10	3	30
9	5	45
8	4	32
7	7	49
6	6	36
5	5	25
4	2	08
3	3	09
2	4	08
1	1	01

$\sum \mathbf{F}=30$ $\sum \mathbf{F}\mathbf{X}=243$

Mean = $\bar{x} = \frac{\sum \mathbf{f}\mathbf{X}}{\sum \mathbf{f}}=\frac{243}{30}=8.1$

The Researcher sometimes obtained large scores and may decide to group them before the calculation of mean. That remind you about the discussion in chapter Two. The following procedures will guide you in calculating the mean of grouped data.

6.3.2Mean of Grouped Data

A Researcher who is interested in grouping data for calculating the mean of a frequency distribution should follow these steps:

*Step 1:*Group the data in class interval (cl)

*Step 2:*Record the frequency (f)

*Step 3:*Calculate the class mark of each class (x)

*Step 4:*Find the products of class mark and frequency (fx).

*Step 5:*Sum up the products resulted in fx

*Step 6:*Divide $\frac{\sum \mathbf{f}\mathbf{x}}{\sum \mathbf{f}}$

The data provided in table 6.3 gives the details on how to compute the mean for group data.

Table 6.2: Scores in Geometry Performance Test

Class Interval	Frequency	Class Mark	FX
61—63	4	92.5	370
64—66	17	97	1649
67—69	36	101.5	3654
70—72	26	106	2756
73—74	7	110.5	773.5
	$\sum \mathrm{f}=90$		$\sum \mathrm{f}\mathrm{x}=9202.5$

By the formula = $\frac{\sum \mathbf{f}\mathbf{x}}{\sum \mathbf{f}}=\frac{9202.5}{90}=102.25$

6.3.3Mean Scores in Research

A Researcher in Mathematics Education who is interested to investigate on factors affecting the teaching and learning of Mathematics in Junior Secondary Schools. The Researcher developed a questionnaire on a 4-point Likert scale to seek for the opinions of Mathematics Teachers over a number of factors.

Strongly Agree (SA) = 4, Agree (A) = 3, Disagree (D) = 2, Strongly Disagree (SD) = 1.

Table 6.3 shows the response of 30 Teachers to the research questionnaire administered by the Researcher.

Table 6.3: Responses of Teachers to the Questionnaire

S/N	Factors	SA	A	D	SD
		(4)	(3)	(2)	(1)
	Attitudes of Mathematics teachers towards teaching of Mathematics	7	15	4	4
	Understanding of Mathematics language by students	10	5	5	10
	Inadequate of lecture method used by all teachers	15	5	5	5

To find out whether attitude of Mathematics Teacher is a contributing factor, the Researcher can adopt this process as follows:

Table 6.4: Responses on Attitude of Mathematics Teachers

Score (x)	F
1	4
2	4
3	15
4	7
Total	30

Mean score of the response on the factor; attitude of Mathematics Teacher could be calculated by following these steps:

*Step 1:*Drawing a frequency distribution Table from Table 6.4

*Step 2:*Find the product of each score with its frequency.

*Step 3:*Sum-up the products.

*Step 4:*Divide the result obtained in Step 3 by the sum of scores.

Adopting the steps above, then the computation of the mean for data in Table 6.5 is shown below

Table 6.5: Mean for Data in Table 6.5

X	F	FX
1	4	4
2	4	8
3	15	45
4	7	28
n = 30		85

$\bar{x} = \frac{\sum \mathbf{f}\mathbf{x}}{\mathbf{n}}=\frac{85}{30}=2.8$

Furthermore, the Researcher can continue to calculate the mean scores for the other factors by the following the steps explained above. Mean scores of the other factors are showed in Table 6.6

Table 6.6: Mean Scores of Factors that Affect Teaching and Learning of Mathematics

S/N	Factors	Mean Scores
	Attitudes of Mathematics teachers towards teaching of Mathematics	2.8
	Understanding of Mathematics language by students	2.5
	Inadequate of lecture method used by all teachers	3

In order to answer the research question asked that what factors affect the teaching and learning of mathematics in J. S.S based on the perception of mathematics teacher.

We need to draw benchmark for comparing the mean score, since the mean on this scale is given as :

$$\frac{1+2+3+4}{4}=2.5$$

Decision can be taken by the Researcher:

1. Retain as factor if the $\bar{x} \geq 2.5$
2. Reject as a factor if the $\bar{x} < 2.5$

By this benchmark, the Researcher will conclude that all the three factors affect teaching and learning of Mathematics in Junior Secondary Schools.

6.4Median

This is the score in the middle of the distribution, dividing the collection of scores into two equal-sized groups. The scores must be put in either ascending or descending order to get the median.

However, when the scores are ordered in order of their size, the median for an even number of points is the average of the middle score. The median serves as the middle score when there are an odd number of points.

Major Properties of Median

It is another measure of central, which the score lies. The major properties of median are:

1. It is only one median for any data set
2. The medians cannot add to find the median of the complete data set
3. Extreme measurement does not affect the median score
4. Median score in group data is stable, even when the data organized into different classes.

6.4.1Median for Ungrouped Data

Determine the median of the sets of scores in a test

1. 0, 7, 5, 13, 8, 9, 6, 1, 2
2. 7, 4, 5, 3, 1, 2, 6, 7, 4, 8

Solution

1. 0, 1, 5, 6, 7, 8, 9, 13, 2

The middle number may be calculated as 6. 6 is the median.

1. 1, 2, 3, 4, 4, 5, 6, 7, 7, 8

Here we have 4 & 5 as middle number then the median is $\frac{4+5}{2}=\frac{9}{2}=4.5$

To find the median of ungrouped data it is easy to use the following steps:

*Step 1:*Rearrange the score in ascending or descending order

*Step 2:*Counting equal number of scores from each end then the middle score in the median, but in a case number of scores in even then two middle scores area picked.

*Step 3:*Calculate the average of these middle scores, the result is the median as illustrated in above example 'b'.

When the researcher obtained large number of scores this will be described in 6.4.2

6.4.2Median for Grouped Data

To find the median of grouped data, it is slightly more difficult when compared with that of ungrouped dat. Since the real values of the scores are unknown, you can only say the median occur in particular class interval and you cannot locate it within the interval.

To calculate the median in this case, you should use the formula:

\mathrm{M}\mathrm{e}\mathrm{d}\mathrm{i}\mathrm{a}\mathrm{n}=\mathrm{L}+\ \mathrm{F}\right)\mathrm{C}}{\mathrm{f}}

Where L = lower class boundary of the class interval consisting of the middle value.

F = Total number of scores below L

f = Frequency of the median class

C = Class interval size

N = Number of scores

In order to find the median with large number of scores, the data in table 6.8 can be used for illustration.

Table 6.7: Cumulative Frequency Distribution of Score

Class Interval	F	Cf
29—31	8	28
26—28	5	20
23—25	3	15
20—22	2	12
17—19	4	10

14—16	6	6

Total (N) = 28

Applying the formula to obtain the median

$$Median = L + \frac{\left(\mathrm{N}/2-\mathrm{F}\right)\mathrm{C}}{\mathrm{f}}$$

Where L = 22.5

N = 28

F = 12

f = 3

C = 3

Substituting in the total formula, then we have:

$$Median = 22.5 + \frac{\left(\mathrm{N}/2-\mathrm{F}\right)3}{3}$$

$$= 22.5 + \frac{2\times 3}{3}$$

= 22.5 + 2

= 24.5

The median as one measure of location is only useful when the distribution of scores is skewed. At that particular case the median serves very useful. However, the median has its own limitation because it is largely considered the mid-point of the distribution, where it lies without concerning for other scores. It is not useful for calculation of higher statistics but better the mode.

To obtain the median of grouped data the following procedure can be adopted.

Step 1: Prepare a cumulative frequency distribution table.

Step 2: Determine half of the distribution as the case may be.

Step 3: Locate the cumulative frequency where the median lies.

Step 4: Find the lower boundary of the score that contains the median.

Step 5: Apply the formula: $Median = L + \frac{\left(\mathbf{N}/2-\mathbf{F}\right)\mathbf{C}}{2}$

6.5Mode

The mode is the highest frequency that occur in a distribution or the most observed score in a set of data. The calculation of mode in data is easy and simple for example, consider this distribution of scores in a test:

Score (X)	20	15	12	10	9
Frequency (F)	5	7	2	3	4

The score that is most typical is 15 because it has the highest frequency, which is 7. It is important to recognize that a distribution of scores may have more than one mode. When distribution of scores indicates one mode is known as unimodal, likewise when distribution have two modes is said to be bimodal. In case, it is more than two modes is multi-modal in nature. The following referred as major properties of mode.

Major Properties of Mode

1. It is the highest frequency in a distribution.
2. Sometimes the mode can be more than one.
3. Extreme measurement does influence the mode.
4. It cannot be combined to find the mode of the complete data set.
5. It can be applied for both quantitative and qualitative data.

It is important to note the mode as a measure of central tendency has limitations. One of the limitations is that mode is not necessarily the representation of other values in the distribution. Another short coming of the mode it cannot be used in the calculation of other statistics. Thus, with these limitations, it is better for a researcher to try other computation of higher statistics.

6.5.1Mode for Ungrouped Data

Let's see how to identify modes in the following cases.

Example 6.1

In the distribution below, identify the mode:

10, 20, 11, 35, 20, 15, 23, 25, 20, 12, 19, 20.

By observation, you will discover that 20 is repeated 4 times. The mode of score is 20.

Example 6.2

Determine the mode in the frequency table below:

X	20	19	18	17	16	15	14	13
F	7	3	5	6	7	6	2	3

Again, by observation, you will discover that 20 and 16 has the highest frequency of the value 7 in the distribution. Therefore, the distribution has two modes in nature, the modes are 20 and 16.

6.5.2 Modes for Grouped Data

In order to calculate mode of grouped data, the formula below can be applied

X =\\mathrm{L}+\\frac{\\left(\\mathrm{\\DifferentialD }^{1}\\right)^{\\mathrm{c}}}{\\mathrm{d}^{1}+\\mathrm{d}^{2}}

Where *L* = the exact lower limit of the modal class

d^1 = difference between frequency of modal class and frequency of the class before the modal class.

d^2 = difference between frequency of modal class and frequency of the class after the modal class.

c = the class size.

Example 6.3

Calculate the mode in the frequency distribution table provide below:

Table 6.9: Scores in Distribution

Class Interval	Frequency
75—79	2
70—74	2
65—69	7
60—61	9
55—59	11
50—54	6

1. By inspection, the modal class is 55—59
2. Applying the formula \\mathrm{L}+\\frac{\\left(\\mathrm{\\DifferentialD }^{1}\\right)^{\\mathrm{c}}}{\\mathrm{d}^{1}+\\mathrm{d}^{2}}

Where L = 54.5, C = 5, d^1 = 11—6 = 5, d^2 = 11—9 = 2

1. X = 54.5 + \\frac{\\left(5\\right)^{5}}{5+2}=54.5+\\frac{\\left(5\\right)^{5}}{7}

= 54.5 + 446.43

= 500.93

6.6 Assumed Mean (A.M)

Computation of mean can be carryout using the assumed mean. If the data collected by researcher consist of large items. It is a short-cut method for calculating the mean. This method involves the use of an arbitrary value from the list of items considered for the purpose of calculating the arithmetic mean is known as Assumed Mean (A.M)

A number is very close to the middle score is usually suggested to avoid ambiguity. The deviation (i.e the assumed mean) subtracted from each of the score denoted by the letter 'u' while the mean of all the deviations recorded is known as mean deviation.

The actual arithmetic mean of the distribution is sum of the assumed mean and deviation

i.e. Actual Mean = Assumed Mean + Mean Deviation

the formula is given as:

\\overline{x}=\\mathrm{A}+\\frac{\\Sigma \\mathrm{f}\\alpha }{\\Sigma \\mathrm{f}}

Where A = Assured Mean

α = Deviation from the Assumed Mean

f = frequency

$\overline{x}$ = Arithmetic Mean

Example 6:4

A frequency table showing scores of nine students in achievement test

26,16,22,43,12,19,14,35,13

Using an assumed mean of 14 to calculate the mean score of the distribution.

Solution

Arrange the scores in ascending order

Table 5.10: Data for Test Scores

Scores (x)	Deviations: d = x—AM
12	12—14 = −2
13	13—14 = −1
14	14—14 = 0
16	16—14 = 2
19	19—14 = 5
22	22—14 = 8
26	26—14 = 12
35	35—14 = 21
43	43—14 = 29
	$\Sigma d=74$

Assumed Deviation $=\frac{\Sigma d}{n}=\frac{79}{9}=8.2$

Actual Mean = Assumed Mean + Assumed Deviation

$=14+8.2$

$=22.2$

Example 6.5

The table below indicates the masses of a number of students in a class. Calculate the Actual Mean using the Assumed Mean as 62.

Table 6.11: Masses of Students

Masses (kg)	48	51	54	62	65	68	72
Frequency	7	4	3	5	1	4	6

Solution

Let prepare the require table for the distribution

Table 6.12 Data obtained as Masses of Students

Masses (kg)	Dev. ($\propto$)=- $\mathbf{A}\mathbf{M}$	$\boldsymbol{F}\boldsymbol{r}\boldsymbol{e}$... ($\boldsymbol{f}$)

51	51—62 = −11	4	−41
54	54—62 = −8	3	−24
62	62—62 = 0	5	0
65	65—62 = 3	1	3
68	68—62 = 6	4	24
72	72—62 = 10	6	60
	A.M = 62	$\sum \boldsymbol{f}=30$	$\sum \boldsymbol{f}\boldsymbol{d}=-72$

Assumed $=\frac{\sum fd}{\sum f}=\frac{-72}{30}=-2.4$

Actual Mean $=A.M+A.D=62-2.4=59.6$

Coded Factor

When the distribution data in a grouped data is large, coded factor is used with the addition of the assumed Mean. The coded is the class size (class width) and denoted by V. Itis obtained through the class width to divide each of the deviation.

The table below shows the distribution weight of students in a class.

Table 6:13 Weight (kg) of Students

Weight (kg)	No. of students
10—19	6
20—29	13
30—39	10
40—49	6
50—59	11
60—69	1
70—79	3

Using an assured Mean of 39.5, calculate the Mean weight of the distribution

Solution

A frequency table showing the different columns and their values is constructed for the distribution as required.

Table 6.14: Distribution Data of Students' Weight.

Weight	Class Mark	Frequency	$\boldsymbol{d}=\boldsymbol{x}\hbox{--}\boldsymbol{A}.\boldsymbol{M}$	\\bold... {\\boldsy...
10—19	14.5	6	–25	–2.5
20—29	24.5	13	–15	–1.5
30—39	34.5	10	–5	–0.5
40—49	44.5	6	5	0.5
50—59	54.5	11	15	1.5
60.69	64.5	1	25	2.5
70—79	74.5	3	35	3.5
		$\sum \boldsymbol{f}=50$		

Note that the class size (c) = 10

Also,

$\mathrm{A}=39.5$

$\sum f=50$

$\sum fd=\hbox{--}70$

$\sum fu=\hbox{--}7$

Then compute actual mean using Assumed Mean:

Therefore, $\overline{x}=A+\frac{\sum fu}{\sum f}$

$=39.5+\frac{\hbox{--}70}{50}$

$=39.5\hbox{--}1.4$

$=38.1kg$

Alternatively using the Assumed mean and coded factor

$\overline{x}=A+\left(\frac{\sum fu}{\sum f}\right)^{c}$

$=39.5+\left(\frac{\hbox{--}70}{50}\right)^{10}$

$=39.5+\left(0.14\right)10$

$=38.1kg$

Mid Range

It is another form of calculating average that is sought from set of Data. Midrange is the addition of the highest score divided by two. For example, the following set of scores in achievement test are:

22,24,25,28,29,32,47and60.

Highest score = 60

Lowest score = 22

Midrange = $\frac{60+22}{2}=41$

6.7Coding Method for Computing Mean

When researcher has large values of variable and computation of the mean used become difficult, and tedious, the coding method can be used. This is done by converting the x-values into simpler values for the computation, and then later convert it back again.

This is carryout subtracting or adding from each original, if that is possible, then divide or multiply these new values which are easily manageable. Then find the mean of the x-values \\overline{x} and by employing a suitable decoding formula to obtain\\overline{x}. An example will be given to show the method in detail.

To apply the coding method to find mean score, the following steps can be adopted.

*Step I:*Choose a convenient value of x (i.e., near the middle of the range)

*Step II:*Subtract these values chosen from every other value of x and recorded it in column 2.

*Step III:*Then convert the values into an even simpler from by dividing the values in column 2 by chosen value. Record the result in column 3.

*Step IV:*Add all class frequencies, write summation in column 4

*Step V:*find the product of values in each frequency. Record the result in column 5.

*Step VI:*Using the column 4 and 5 to find the mean.

Now, let us illustrate the procedures to find the mean scores using coding method.

Example 6:8

A research measure in millimeters of 50 bolts gave the following frequency distribution in Table 6.15

Length x (mm)	20.2	20.4	20.6	20.8	21.0	21.2	21.4
Frequency (f)	2	6	10	15	9	7	1

Solution

\\boldsymbol{L}\\boldsymbol{e}\\boldsymbol{n}\\boldsymbol{g}\\boldsymbol{t}\\boldsymbol
(\\boldsymbol{m}\\boldsymbol{m})

Units is 0.2mn

$\boldsymbol{x}_{\boldsymbol{u}}=\frac{\boldsymbol{x}-20.8}{0.2}$		$(\boldsymbol{f})$	[illegible]
20.2	–0.6	–3	[illegible]
20.4	–0.4	–2	[illegible]
20.6	–0.2	–1	[illegible]
20.8	0	0	[illegible]
21.0	0.2	1	[illegible]
21.2	0.4	2	[illegible]
21.4	0.6	3	[illegible]

$\therefore \quad \overline{\boldsymbol{x}}=\frac{-2}{50}=-0.04$ [illegible]

If $\overline{x}_{u}=-0.04$, we can return back to the original unit of x

Decoding

In the coding step, the last step was divided by 0.2, the reverse is to multiply by 0.2 return to the correct units of$(x-20.8)$. Therefore:

$\overline{x}-20.8=-0.04\times 0.2=-0.08$

Add 20.8 to both sides, you have

$\overline{x}=20.8-0.8=20.72=\overline{x}=20.72$

6.8General Observation about Measure of Central Tendency

1. The mean can be affected by extreme values in the distribution but not the Median. For example: Take look into this set of scores.

$x\colon 6,8,9,10$

$\overline{x}=\frac{6+7+8+9}{5}=\frac{40}{5}=8.$

While the Median=8i.e, Mean = Median.

If the set of scores is changed to be 6, 7, 8, 9, 20

The mean = $\overline{x}=\frac{6+7+8+9}{5}=\frac{50}{5}=10$ but the Median remains 8.

1. The mode may not exist in a distribution and is not always unique, but mean median always exists and has uniqueness.

\\boldsymbol{E}\\boldsymbol{x}\\boldsymbol{a}\\boldsymbol{m}\\boldsymbol{p}\\boldsymbol
The set of scores 6, 7, 8, 9, 10 has no Mode 6, 6, 7, 7, 9 is bimodal. And the mode is not unique because it may have more than one and two.

1. The addition of the deviation from the mean always equal zero
2. The four basics on measure of location by a constant lead to the following definitions.
3. Addition and Subtraction

For any distribution, if a constant is added to (or subtracted from) each observation, the corresponding measure of central tendency changes by the same value or score.

\\mathbf{E}\\mathbf{x}\\mathbf{a}\\mathbf{m}\\mathbf{p}\\mathbf{l}\\mathbf{e}\\colon
Look at the following set of scores in a test: 3, 8, 9, 10, 10.

Original data	Plus 3	Minus 3
3	3	0
8	6	5
9	11	6
10	13	7
10	13	7
40	55	25

Mean =	8	11	5
Median =	9	12	6
Mode =	10	13	7

b. Multiplication or Division

If each observation is multiplied (or divided) by a constant, the corresponding measure of central tendency must also be multiplied or divided by the same value or score.

\\boldsymbol{E}\\boldsymbol{x}\\boldsymbol{a}\\boldsymbol{m}\\boldsymbol{p}\\boldsymbol
Using the Data above

Original Data	Multiply by 3	Divide by 3
3	9	1
8	24	2.7
9	27	3
10	30	3.3
10	30	3.3

40	120	13.3	
Mean \\overline{X}=	5	24	2.7
Median=	9	27	3
Mode =	10	30	3.3

Student Activity

1. The following data are the summary of response of 50 secondary school teachers to a questionnaire on factors affect teaching efficiency.

S/N	Factors	Strongly Agree 4	Agree 3	Disagree 2	Strongly Disagree 1
	Teacher's qualification	20	10	15	5
	Appropriateness of teaching instruction	5	15	25	5
	Lack of Textbooks	30	10	5	5
	Using of Learning Aids	20	20	5	5
	Lack of interest by students	25	10	5	5

Using the data above:

1. Calculate the teachers' mean responses for each factor.
2. On the basis of the statistics calculated, rank the factors.
3. Analyse the data to identify which factor affect the teaching in primary schools
4. Find the mean, median and mode for the data obtained in the frequency table.

Class Interval	Frequency
34.9—39.9	5
39.9—44.9	8
44.9—49.9	13
49.9—54.9	5
– 59.9	1

1. Enumerate and explain what you know about measures of central tendency.
2. List major properties of mean, mode and median.
3. Compute the mean, mode median for the following data.
4. 27, 21, 19, 32, 33, 25, 27.
5. 4, 3, 1, 8, 8, 1, 4.

Gams (mm)	Frequency
40—49	7
50—59	10
60—69	5
70—79	8
80—89	18
90—99	10
100—109	12

1. Using the assumed mean of 84.5, calculate the gam size of the distribution.

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CHAPTER SEVEN

MEASURE OF VARIABILITY

7.1 Introduction

In the previous chapter, you have learnt Measure of Central Tendency. Measures of Central Tendency do not provide enough information about description of the data. Measures of Central Tendency only describe a distribution in terms of mean score of the highe st frequency score but no en

ough information about desc

ription of the data. Sometimes mean and median may be the same for a data without informing us whether they are dispersed or spread. For instance,

The scores 5, 6, 7, 8, 9 has a mean of 7 and 1, 3, 5, 8, 13 has a mean of 7. The two sets recorded the same mean but is well-informed that the first set is more rightly arranged around comparing to the second set. Therefore, there is the need for a measure to inform us how the score disperses or spread about the mean. That is why we need an index that describe the variability scores in a distribution.

Measures of Variability are also known as Spread or Dispersion Measures. This measurement depicts how far away from the mean different points in a distribution are. The range, quartile, variance, and standard deviation are among the measures of dispersion.

7.2Objectives

At the end of this chapter, you should be able to:

1. describe the measures of variability, range, quartile, variance, and standard deviation.
2. calculate range, quartile, variance, and standard deviation.

7.3Range

The easiest way to measure dispersion is to look at a distribution's range. The disparity between the greatest and lowest scores in the data is what we mean by this. Sometimes a distribution's score range is either inclusive or exclusive. When the difference between the upper border of the interval comprises the lowest score, the range is said to be inclusive. The exclusive range, however, is the variation in a distribution's top and lowest scores.

Range is not a stable indicator of the nature of the spread of the measures around the central value. It is a method of determining spread as it takes into account of only the two extremes in a distribution.

7.3.1Range for Ungrouped Data

To demonstrate on how to determine a range of data, which is ungrouped. Let us look in example 7.1.

Determine the range of the scores in mathematics achievement test: 76, 90, 71, 95, 65, 60, 71, 75

The highest score = 95

The lowest score = 60

Range = 95—60 = 35

7.3.2Range for Grouped Data

For a grouped data, individual measurement is not known for that fact, the range is considered to be the difference between the upper limit of the last class interval using the distribution of scores:

50, 25, 27, 30, 50, 45, 22, and 34. Here, the highest boundary limit 50.5 and the lowest boundary limit is 21.5.

Range = 50.5—21.5 = 29. This approach of calculating range is defined as

Range = H—L + 1

= 50—22 + 1 = 29

7.4Percentile

Another measure of dispersion is carried out by the use of percentile. The P^{th} percentile of a set of "n" measurement arranged the scores in order of magnitude as it provides estimation value at most P% of the measurement below it and at most (100—P) % of the measurement above it. Where P is a value between 0 and 1.

To describe the results of achievement test scores and the ranking of a person in comparison to all other students who took a particular test is achieved through the use of percentiles.

7.5Quartiles

These values create four equal halves from a given data collection. that each component stands for $\frac{1}{4}\mathrm{t}\mathrm{h}$ of the population or sample. That is to say, the number of scores in any one of the four sections equals the number of scores in any one of the three remaining parts. First quartile data may be used to illustrate this Q_1, second quartile $Q_{2,}$ and the quartile Q_3.

7.5.1Computation of the Quartiles.

To compute quartiles in s grouped data, the below formula can be applied

$$\mathbf{Q}_{\mathbf{i}}=\mathbf{L}+\frac{\left(\mathbf{i}\left(\frac{\mathbf{N}}{4}\right)-\mathbf{C}\mathbf{f}\mathbf{b}\right)\mathbf{C}}{\mathbf{f}\mathbf{w}}$$

Where i = 1, 2, 3, i.e., quartiles

N = $\Sigma \mathrm{f}$ is the sample size

L = Lower class boundary of the quartile class

Cfb = Cumulative frequency below the quartile class

fw = frequency of the quartile class

C = Class interval size

Let us now, illustrate the calculation of the first quartile (Q_1), the third quartile (Q_3) and semi-interquartile range.

Example 7.2

Determine Q_1 and Q_3 In the given distribution

Class Interval	F	Cf
60—64	1	34
55—59	2	33
50—59	2	31
45—49	5	29*
40—44	8	24
34—39	6	16
30—34	4	10*
25—29	3	6
20—24	2	3
15—19	1	1
	34	

Now let us adopt the following procedures:

Step 1: Determine the cumulative frequency (cf)

Step 2: Divide 34 by 4 = $34 \div 4 = 8.5$

Step 3: Use the formula $\mathbf{Q}_{\mathbf{i}}=\mathbf{L}+\frac{\left(\mathbf{i}\left(\frac{\mathbf{N}}{4}\right)-\mathbf{C}\mathbf{f}\mathbf{b}\right)\mathbf{C}}{\mathbf{f}\mathbf{w}}$

For the first quartile (Q_1) = 8.5 is between the class 30—34

L = 29.5, fw = 4, Cfb = 6

$\mathbf{Q}_{\mathbf{i}}=\mathbf{L}+\frac{\left(\mathbf{i}\left(\frac{\mathbf{N}}{4}\right)-\mathbf{C}\mathbf{f}\mathbf{b}\right)\mathbf{C}}{\mathbf{f}\mathbf{w}} = 29.5+\frac{\left(8.5\hbox{--}6\right)5}{4}=29.5+\frac{\left(2.5\right)5}{4}$

= 29.5 + 3.13

= 32.63

For third quartile (Q_3) lies between 45—49

L = 44.5, fw = 8.5, Cfb = 24

$\mathrm{Q}_{3} = 44.5+\frac{\left(3\times 8.5\hbox{--}24\right)5}{5}=44.5+\frac{\left(1.5\right)5}{5}$

= 44.5 + 1.52

= 46.02

To determine the interquartile range in the distribution data is to find the differences between the first quartile Q_1 and the third quartile Q_3. By the computation provided through the working example above.

Q_3 = 46.02—32.63 = 13.39

While the semi-interquartile range is the half value of the inter-quartile range. Semi-interquartile range is also known as quartile deviation, and it is computed by the formula:

$\frac{\mathbf{Q}_{3}-\mathbf{Q}_{1}}{2}$

Semi-interquartile range = $\frac{46.02-32.63}{2}$ = 6.70

7.6 Variance

Another form of Measure of Variability is Variance. The variance Is calculated by taking the mean of the sum of squared deviation of individual score from their average. It is also known as mean square or mean squared deviation. The variance is denoted by the lower-case Greek symbol δ^2 and it is defined by the formula:

$$\mathbf{\delta}^2 = \frac{\mathbf{\Sigma}\mathbf{f}\left(\mathbf{x}-\overline{\mathbf{x}}\right)^{2}}{\mathbf{N}\hbox{--}1}$$

This formula given above is known as definitional formula which is used for computing δ^2 when the sample size is small. It can also be used for large sample size, but it is tedious. For computational purposes, the formula below is usually preferred

$$S^2 = \frac{\sum \mathbf{f}\mathbf{x}^{2}-\frac{\left(\sum \mathbf{f}\mathbf{x}\right)^{2}}{\mathbf{n}}}{\mathbf{n}-1}$$

The two letters S^2 and δ^2 for sample variance and population variance respectively.

7.6.1 Computation of Variance for Ungrouped Data

To calculate for variance of ungrouped data with a finite set of score, you can apply the following procedure.

For instance, 9 students have the following scores in a test.

9, 4, 5, 7, 2, 6, 8, 7, 5.

Step 1: Compute the mean score of distribution.

$\frac{9+4+5+7+2+6+8+7+5}{9}=\frac{53}{9}=5.9$

Step 2: Compute the difference of each score from the mean and square the result of each.

$(9—5.9)^2 + (4—5.9)^2 + (5—5.9)^2 + (7—5.9)^2 + (2—5.9)^2 + (6—5.9)^2 + (8—5.9)^2 + (7—5.9)^2 + (5—5.9)^2$

$(3.1)^2 + (- 1.9)^2 + (- 0.9)^2 + (1.1)^2 + (- 3.9) + (0.1)^2 + (2.1)^2 + (1.1)^2 + (0.9)$

9.61 + 3.61 + 0.81 + 1.21 + 15.21 + 0.01 + 4.41 + 1.21 + 0.81 = 36.89.

Step 3: find the mean of these scores and that gives the variance $\frac{36.89}{9}=4.10$

7.6.2 Computation of Variance for Grouped Data

To compute for variance in grouped data both definitional formula and working formula can be used by a researcher, who desires, but the definitional formula is too tedious and may be greatly affected by rounding up errors. Both have the same procedure as in ungrouped data but the only difference, the class mark of each class is used for the score (x).

Let us illustrate the computing of sample variance of test score, using Definition Formula. Data provided below is the test scores of 50 Students in Geography test.

Table 7.1 Distribution Scores for 50 Students in Geography Test.

A Class Mark (X)	B Frequency (F)	C Fx	D x - $\overline{\mathbf{x}}$	E (x - $\overline{\mathbf{x}})^2$	F F
10	3	30	4.58	20.98	62.94
9	4	36	3.58	12.82	51.28
8	3	24	2.58	6.66	19.98
7	8	56	1.58	2.50	20.00
6	10	60	0.58	0.34	3.40
5	5	25	−0.42	0.18	0.90
4	5	20	−1.42	2.02	10.10
3	2	6	−2.42	5.86	11.72
2	4	8	−3.42	1.70	46.80
1	6	6	−4.42	19.54	117.24

$n = 50$ $\mathbf{\Sigma}\mathbf{f}\mathbf{x} = 271$ $\mathbf{\Sigma}$F(x - $\overline{\mathbf{x}}$ 344.36

$\overline{\mathbf{x}}=5.42$

To determine sample variance by using definitional formula, the following steps should be adopted:

Step 1: Determine the mean, as $\overline{\mathbf{x}}=\frac{\mathbf{\Sigma}\mathbf{f}\mathbf{x}}{\mathbf{n}}$ which is $\frac{271}{50}=5.42$

Step 2: Find the deviation score and write the results in Column B (x - $\overline{\mathrm{x}}$).

Step 3: Square the deviation scores obtained and record in Column C, i.e., (x - $\overline{\mathrm{x}})^2$.

Step 4: Find the product of $F(x - \overline{\mathrm{x}}\right)^2$ and write the results in Column D.

Step 5: Sum up the product of $F(x - \overline{\mathrm{x}}\right)^2$ and record the results in Column E, which is $\Sigma F(x - \overline{\mathrm{x}}\right)^2 = 344.36$

*Step 6:*Compute the variance given as $\frac{\mathbf{\Sigma}\mathbf{f}\left(\mathbf{x}-\overline{\mathbf{x}}\right)^{2}}{\mathbf{n}\hbox{--}1} = \frac{344.36}{50\hbox{--}1} = \frac{344.36}{49} = 7.03 \approx 7.0$

The working formula that is much easier is illustrate using the data in Table 7.4.

Table 7.1 Distribution Scores for 50 Students in Geography Test.

A Class Mark (X)	B Frequency (F)	C FX	D X^2	E FX^2
10	3	30	100	300
9	4	36	81	324
8	3	24	64	192
7	8	56	49	392
6	10	60	36	360
5	5	25	25	125
4	5	20	16	80
3	2	6	9	18
2	4	8	4	16
1	6	6	1	6

$\mathbf{\Sigma}\mathbf{F} = 50 \quad \mathbf{\Sigma}\mathbf{F}\mathbf{X} = 271 \quad \mathbf{\Sigma}FX^2 = 1813$

$\frac{\mathbf{\Sigma}\mathbf{F}(\mathbf{X})^{2}}{\mathbf{n}}=\frac{271}{50}=1468.82$

To work out variance using working formula, the following procedure should be adopted

Step 1: Draw a frequency distribution of the scores

Step 2: Find the product of each score with its frequency and write the result in Column B (FX)

Step 3: Add the values of FX and divide by n, that is $\frac{\Sigma \mathrm{F}(\mathrm{X})^{2}}{\mathrm{n}}=1468.82$

Step 4: find the square of each score (X) and record in column D.

Step 5: Find the product of each (X^2) with its frequency and write the results in column E.

We have $\Sigma FX^2 = 1813$

Step 6: Substitute all the values obtained in the formula, that is $(\Sigma FX)^2$, ΣFX^2

$$S^2 = \frac{\sum \mathbf{f}\mathbf{x}^{2}-\frac{\left(\sum \mathbf{f}\mathbf{x}\right)^{2}}{\mathbf{n}}}{\mathbf{n}-1}$$

$$S^2 = \frac{1813\hbox{--}1468.82}{50-1}=\frac{344.18}{49}=7.02\approx 7.0$$

The two formulas, definitional formula and working formula were used to obtain the same values of variance. You can discover that using working formula is simpler and can yield more accurate result that using definition formula that is tedious and affected with rounding up errors.

7.7Mean Deviation

It is simply mean the deviation by which individual scores in a distribution differ from the mean. However, summing up all the mean deviation for each score to the average always zero. For example, the scores are: 95, 90, 85, 80, and 75.

\\mathrm{A}\\mathrm{v}\\mathrm{e}\\mathrm{r}\\mathrm{a}\\mathrm{g}\\mathrm{e}=\\fra
{5}=\\frac{425}{5}=85

If the average is 85, then deviation will be

$95\hbox{--}85=10$

$90\hbox{--}85=5$

$85\hbox{--}85=0$

$80\hbox{--}85=\hbox{--}5$

$75\hbox{--}85=\hbox{--}10$

It is good to note that, the mean is always zero, because the negative number cancel out the positive numbers.

7.8Standard Deviation

The standard deviation is adopted by the letter 'S" for sample of data while that of the population is represented by the lower-case Greek letters δ (sigma). The standard deviation is simply the variance's square root. It is a generally reliable measure of variability and is regarded as the best indicator of dispersion. A low standard deviation suggests that the data points tend to be extremely near to the average, whereas a high standard deviation indicates that the points are dispersed over a wide range of values. It reflects how much departure from the mean there is value. It gives an idea of how close the entire set of data is to the mean value.

Therefore, variance of ungrouped data which is $\frac{53}{9}=5.9$, standard deviation will read as

$$\sqrt{\frac{53}{9}}=\sqrt{5.9} = 2.429$$

7.8.1Standard Deviation for Grouped Data

When the data is grouped in the class intervals, the researcher should use a modified formula either definitional or working formula. The definitional formula given as: $S = \sqrt[]{\frac{\mathbf{\Sigma}\mathbf{f}\left(\mathbf{x}-\overline{\mathbf{x}}\right)^{2}}{\mathbf{n}\hbox{--}1}}$ and that of population is defined as: $\mathbf{\delta}=\sqrt[]{\frac{\mathbf{\Sigma}\mathbf{f}\left(\mathbf{x}-\overline{\mathbf{x}}\right)^{2}}{\mathbf{N}}}$

*Step 1:*Write down the scores and their frequencies in Column into A and B.

*Step 2:*Find the mean of distribution $\frac{\mathbf{\Sigma}\mathbf{f}\mathbf{x}}{\mathbf{n}}$ and write it down in column C.

*Step 3:*write down $x - \overline{\mathbf{x}}$ note it down in Column D.

*Step 4:*Record square of the deviation record it down in column E, i.e., $\left(\mathbf{x}-\overline{\mathbf{x}}\right)^2$

*Step 5:*Find the product of $(\mathbf{x}-\overline{\mathbf{x}})^2$ and enter it down in column F.

Step 6:Find the sum of $\mathbf{\Sigma}\mathbf{f}(\mathbf{x}-\overline{\mathbf{x}})^2$.

*Step 7:*Divide $\mathbf{\Sigma}\mathbf{f}(\mathbf{x}-\overline{\mathbf{x}})^2$ by n—1 to obtain the variance.

Step 8: Compute the square root of the variance to obtain the standard deviation.

Let us use the data in Table 7.1 to calculate the standard deviation using the definitional formula.

For n = 50 and $\Sigma \mathrm{f}(\mathrm{x}-\overline{\mathrm{x}})^2$ = 344.36, substitute in the formula as

$S = \sqrt[]{\frac{\mathbf{\Sigma}\mathbf{f}\left(\mathbf{x}-\overline{\mathbf{x}}\right)^{2}}{\mathbf{n}\hbox{--}1}} = \sqrt[]{\frac{344.36}{50\hbox{--}1}} = \sqrt[]{\frac{344.36}{49}} = \sqrt[]{7.0} = 2.65$

Another modified formula for computing the standard deviation is working formula, stated as: $S = \sqrt[]{\frac{\sum \mathbf{F}\mathbf{X}^{2}-\frac{\left(\sum \mathbf{F}\mathbf{X}\right)^{2}}{\mathbf{n}}}{\mathbf{n}-1}}$

Where all the terms hold as for the variance. When a Researcher desired to compute for standard deviation, the following procedure should be applied.

Step 1: Develop a frequency distribution table

Step 2: Find the product of FX i.e., multiplying the score and its frequency

Step 3: Add up the (FX) values to obtain ($\sum \mathrm{F}\mathrm{X}$), square it and divide by n. That is $\frac{\left(\sum \mathbf{F}\mathbf{X}\right)^{\mathbf{n}}}{\mathbf{n}}$

Step 4: Each score should be squared. That is (X^2)

Step 5: Record the product of (X^2) by its frequency and sum all the values. That is $\sum \mathbf{f}\mathbf{x}^{2}$

Step 6: Calculate the variance by substituting in the working formular as: $S^2 = \frac{\sum \mathbf{F}\mathbf{X}^{2}-\frac{\left(\sum \mathbf{F}\mathbf{X}\right)^{2}}{\mathbf{n}}}{\mathbf{n}-1}$

Step 7: Obtain the standard deviation by finding the square root of variance. $S = \sqrt[]{\frac{\sum \mathbf{F}\mathbf{X}^{2}-\frac{\left(\sum \mathbf{F}\mathbf{X}\right)^{2}}{\mathbf{n}}}{\mathbf{n}-1}}$

Let us use the data in Table 7.2 to compute the standard deviation using the working formula,

For $n = 50$, $(\sum \mathrm{F}\mathrm{X}) = 271$, $\sum \mathrm{F}\mathrm{X}^{2}=1813$

Substitute in the above formula to obtain standard deviation.

$$S = \sqrt[]{\frac{\sum \mathbf{F}\mathbf{X}^{2}-\frac{\left(\sum \mathbf{F}\mathbf{X}\right)^{2}}{\mathbf{n}}}{\mathbf{n}-1}}$$

$$S = \sqrt[]{\frac{1813\hbox{--}\frac{\left(271\right)^{2}}{50}}{50-1}} = \sqrt[]{\frac{1813\hbox{--}1468.82}{50-1}} = \sqrt[]{\frac{344.18}{49}} = \sqrt[]{7.0} = 2.65$$

The variance and standard deviation have many merits over other measures of variability. These include the fact that the sample standard deviation is a more accurate estimate of the population parameter than other measure of variability.

The variance and standard deviation are good for the calculation of many types of statistics and also widely used as measures of error.

Student Activity

1. Given the measurements 8, 24, 18, 14, 12, and 4. Compute the range, the variance, and the standard deviation.
2. Consider a set of seven scores say 1, 2, 3, 4, 5, 6, 7.

Calculate:

1. $\sum \mathrm{x}$
2. $\sum \mathrm{x}^{2}$
3. $\Sigma \mathrm{f}(\mathrm{x}-\overline{\mathrm{x}})^2$
4. The scores for achievement test are as follows

64695361605174639380
55595755304070324 34
48354467525347425037

32495843441252395665
68756786212173626650

1. Construct frequency distribution table with class interval 0—9, 10—19, _ _ _
2. Compute the variance and standard deviation.

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CHAPTER EIGHT

MEASURES OF

RELATIVE POSIT

ION

8.1Intro

duction

In

the previous

chapter, you

learnt how to

compute the variance and standard deviation. This chapter will expose you to Measures of Relative Position. This indicates where a score is in the distribution. It permits one to compare the performance of an individual with all other individuals in the same sample, all measured on the same variable. Measures of Relative Position that will be discussed here are Z-scores and T-scores.

8.2Objectives

At the end of this chapter, you should be able to:

1. define Z-scores
2. define T-scores
3. compute Z-scores and T-scores.
4. List and explain the properties of Z-score and T-score.
5. To represent skewedness of a distribution
6. Represent kurtosis of a distribution

8.3Z-Score

It is a straightforward standard score that translates test performance into a raw score above or below the mean, or the amount of standard deviation units. The formula below is used to determine the Z-score:

Z–score $=\frac{\mathbf{X}-\overline{\mathbf{X}}}{\mathbf{S}\mathbf{D}}$

Where X = raw score

$\overline{\mathbf{X}}$ = mean of the raw scores

SD = standard deviation

If this is not taken into account in the test interpretation, the fact that the Z-score is negative because the raw score is less than the mean might have major consequences. As a result, Z-scores are transformed into a common scoring scheme that only accepts non-zero and positive values. Z-scores often fall within a range– 4 to + 4 ($-4< Z< 4$).

8.4T-Score

T–score is linear transformation of Z–score into a higher number or index using the formula. T–score = 10Z + 50

8.5Computation of Z–score and T–score

A test with mean score 40 and a standard deviation was 4. Calculate T–scores of two tests with raw scores of 45 and 30 respectively in the test.

Solution

To compute for T-scores, you must firstly calculate Z-scores for the testee. Then the Z-scores would be converted to the T–score required.

1. The raw score of 45, the Z—score is

Z–score $= \frac{\mathbf{X}-\overline{\mathbf{X}}}{\mathbf{S}\mathbf{D}}$ where X = 45, $\overline{\mathbf{X}}$ = 40, SD = 4

Z–score $= \frac{45\hbox{--}40}{4} =\frac{5}{4}=1.25$ then

T–score = 50 + 10(Z),Z—score = 1.25

= 50 + 10 (1.25)

= 50 + 12.5

= 62.5

1. The raw score of 30, the Z–score is

$Z\text{–score} = \frac{\mathrm{X}-\overline{\mathrm{X}}}{\mathrm{S}\mathrm{D}}$ where $X = 30$, $\overline{\mathrm{X}} = 40$, $SD = 4$

$=\frac{30\hbox{--}40}{4} = \frac{\hbox{--}10}{4}$

$= \hbox{--}2.5$

Now the T–score = 50 + 10 (Z)Z—score = $\hbox{--}2.5$

= 50 + 10 (- 2.5)

= 50—25

= 25

8.5.1Properties of Z-score and T-score

The following are properties (characteristics) of the two scores.

1. Z-Score
2. Range; generally, as $4.0\leq Z \leq 4.0$
3. Mean; $\overline{z}=0$ and lastly,
4. Standard deviation as $S.\mathrm{D}_{Z}=1.00$
5. T-Score
6. Range; generally, as $10\leq T\leq 90$
7. Mean; T=50 and lastly,
8. Standard deviation as $S.\mathrm{D}_{T}=10$

8.6Normal Distribution

Normal Distribution is sometimes known as Gaussian distribution and is a continuous probability distribution, where in values under the curve is always 1 or 100%.

The formula for the normal probability density function seen complicated. You only need to know the population mean and standard deviation when to use it.

If it is represented in graph form, the normal distribution appears as a bell curve. A normal distribution is symmetrical, but all symmetrical distribution is normal. Many naturally occurring phenomenon tend to approximate the normal distribution but in finance, most pricing distributions are not however, perfectly normal.

The formula for a normal distribution is given as:

$$f\left(x\right)=\frac{1}{\delta\sqrt[]{2\overline{\wedge}}}e^{\frac{\hbox{--}1}{2}\frac{(x-\mathrm{\mu})^{2}}{\delta}}$$

Where x= value of the variable & $f\left(x\right)$= the probability function

$\mathrm{\mu}$= The mean

δ = The standard deviation.

$\overline{\wedge}=3.142$ Approximate.

e=2.718 Approximate.

The Normal Distribution curve is a frequency polygon and symmetrical about a point at which the mean, median and mode are all equal. That is, it is symmetrical about a maximum.

0

Many variables form a normal distribution, including physical measures, such as height, distance, weight, and psychological measures, such as intelligence and aptitude. Most variables measurement in education from normal distribution if enough subject is tested.

8.6.1Skewed Distribution

A distribution that is not normal, non-symmetric, then the value of mean, median and mode are not equal and that it is said to skewed distribution Skewedness is the degree of a symmetric and there are of two type's skewedness-positive skewedness and negative skewedness.

1. Positive Skewedness

A distribution skewed to the right that is when the longer tail for the curve occur to the right. The mean is greater than the median which in turn greater than the mode.

mean

median

mode

mean

median

mode

Fig 6.1 Positive Skewed Distribution

In a positive skewed distribution, the mean has the highest value, then the median and followed by mode.

This distribution take place when test is too difficult or not properly rated.

1. Negative Skewedness:

Negative skewedness take place if the longer tail of the curve falls to the left. That is the mean value is of the lowest than followed by the median and mode has the highest value. This can occur where there is too simple test.

mode

median

mean

mode

median

mean

Fig 6.2 Negative Skewed Distribution

The formula for coefficient of skewedness is as:

$$S=\frac{\sum \left(x-\overline{x}\right)^{2}}{\left(\frac{\sum \left(x-\overline{x}\right)^{2}}{N}\right)^{3}}$$

A normal distribution, s=0

Positive skewedness $=\left(+\right)$

Negative skewedness $=\left(-\right)$

8.6.2Kurtosis

Kurtosis is a work derived from Greek word meaning "curved".

It is the degree of Peakedness of a distribution or a measure that indicates the concentration of the score close to the mean. Measurement of the degree of Peakedness of a Distribution is known as coefficient of kurtosis. Computation for coefficient of kurtosis is given as

$$k=\frac{\sum \left(x-\overline{x}\right)^{4}}{\left(\sqrt[]{\frac{\sum \left(x-\overline{x}\right)^{2}}{N}}\right)^{4}}$$

00When the coefficient of kurtosis IS K = 3. It is Mesokurtic. When the coefficient of kurtosis is greater than 3(k>3), the distribution is Leptokurtic, while when it is less than 3 (k<3), it is Platykurtic

Fig 6.3a Fig 6.3bFig 6.3c

Fig 6.3a—Leptokurtic

Fig 6.3b—Platykurtic

Fig 6.3c—Mesokurtic

Student Activity

1. A distribution of percentage scores has a mean 50 and a standard deviation of 15. If two percentage scores from the distribution were converted to T–scores as:
2.
3. 20 and (ii) 80, find the raw scores.

1. A Test's standarda deviation is 5, with a mean score of 50. Determine te T-Scores for two tastes who performed with raw scores of 25 and 10 in the test, respectively.

Scores	Frequency
9—28	9
19—28	1

29—38	1
39—48	10
49—58	25
59—68	25
69—78	28
79—88	15
89—98	10
99—108	1
109—118	1

3. The follwing is the distrobitopm pf the scpres pf 12- SSII students in posttest.

1. In this distribution if you discover that the mean, mode and median are all equal comment.
2. If this distribution is leptokurtic, platykurtic or Mesokurtic. Justify in a single line to explain the reason for your choice.
3. Consider the values of this distribution in the table below

Mean	Median	Mode
70	65	60
14	14	14
50	62	70

In each case how would you describe the distribution of the observation, using the terms symmetric, positively skewed negatively skewed.

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15

CHAPTER NINE

MEASURES OF

ASSOCIATION

9.1Introduct

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n were advanc

ed that permits the Researcher to describe data, compute measurements of relative locations, measures of variability, and measures of central tendency.

You will discover the measurements of link (association) between variables in this chapter. Correlation and regression are connection measures that involve comparing two or more sets of data to determine whether or not they have any common traits. A perfect positive relationship between two or more variables is shown by a value of 1, whereas a perfect negative relationship is shown by a value of –1, and there is no relationship at all by a value of 0.

For example, a Researcher may be interested to determine relationship between Students' Anxiety and Academic Performance in a course, the relationship between Parents' Socio-economic status and Students' Performance, peer-Group influence and Students' performance, among others.

9.2Objectives

At the end of this chapter, you should be able to:

1. demonstrate all types of relationships

2. compute the Pearson 'r' and
3. compute the spearman rho.
4. explain the meaning of correlation
5. list the importance of correlation

9.3Importance of Correlation

The followings are some of reasons why we measure the correlation between different variables.

These are.

1. To determine the causes and effect relationship
2. To find out whether a relationship exist or not
3. To examine whether it is significant
4. To determine the direction of relationship; and
5. To conclude or analyse usefulness of data.

9.4Types of Correlation

Correlation is of different types; linear and non-linear(curvilinear)correlation. The difference between linear and non-linear correlation is rated upon ratio of change between the variables. Linear correlation is type in which the change in one variable leads to change in another variable in the proportion. Consider this example.

A	5	7	9	11	13	15
B	50	70	90	110	130	150

To plot this point on a graph, you will have straight line. Non-linear correlation is a type of relationship in which the change in the value of one variable does not cause a proportional change in the value of the other variables. The change for the two variables is not proportional (i.e. are different).

Positive correlation and negative correlation.

Examples to show relationship between positive and negative correlation.

Positive	**Correlation**
X	**Y**
10	60
9	59
8	58
7	57

6	56
5	55

Negative	**Correlation**
X	**Y**
10	55
9	54
8	53
7	52
6	51
5	50

Zero Correlation

When there is no relationship between two variables then it is said to be zero correlation.

Student	X	Y
1	10	60
2	9	59
3	8	58
4	7	57
5	6	56

From example, it shows that, there is no relationship exists X and Y. The student that obtained highest value in X, has lowest in Y.

All these relationships can be illustrated in the 9.1a, 9.2b and 9.3c.

"Thus, the relationship or correlation with a numerical value that is coefficient of correlation". This indicates the extent or degree of relationship between the variables. Positive one (+1) means perfect positive correlation, negative one (–1) means perfect negative correlation and zero means there is no correlation. Between the variables. These are various method, in calculating coefficient of coefficient of correlation. But only two methods will discuss.

These methods are Pearson's product moment correlation (R) and Spearman (rho).

Product moment method was developed by Karl Pearson. It is denoted by symbol (r). The value of "r" may be regarded as an arithmetic mean of standard scores. While the other method was developed by the British psychologist, Charles Edward Spearman. It is known as Spearman's coefficient of correlation. It is also denoted by P (rho). Rank difference method is used when ordinal scale data I the form of ranks are given. This method is only possible when the number of observations is small. Also, if only individual scores are given and frequently distribution.

9.5Computation of Pearson Product-Moment Correlation Coefficient

The most widely used measure of association is correlation known as Pearson 'r', which was named in Honor of the man Karl Pearson that developed it. There are two methods of computing the Pearson (r); mean deviation and raw score. The result obtained is the index of relationship between two variables, which refers to correlation coefficient.

Let us use mean deviation formula to compute the correlation between two variables (X and Y).

The formula is stated as: \\frac{\\mathbf{\\Sigma }\\left(\\mathbf{X}-\\overline{\\mathbf{X}}\\right)\\left(\\mathbf{Y}-\\overline{\\mathbf{Y}}\\right)}{\\sqrt[]{\\mathbf{\\Sigma }\\left(\\mathbf{X}-\\overline{\\mathbf{X}})^{2}\\mathbf{\\Sigma }\\left(\\mathbf{Y}-\\overline{\\mathbf{Y}})^{2}}}or \\frac{\\mathbf{\\Sigma }\\mathbf{X}\\mathbf{Y}}{\\sqrt[]{(\\mathbf{\\Sigma }\\boldsymbol{X}^{2})(\\mathbf{\\Sigma }\\boldsymbol{Y}^{2})}}

Example 9.1: The Researcher that conducted a study on the influence of problem-solving attitude on academic performance of pre-service teachers in department of Mathematics obtained the following data.

Table 9.1: Problem-solving Attitude and Academic Performance Scores of Pre-service Teachers.

Pre-service Teachers	1	2	3	4	5	6	7	8	9	10	11	12
Problem-solving Attitude Score (X)	11	12	13	13	14	15	16	16	17	18	18	19
Academic Performance Score (Y)	25	40	45	20	35	30	40	45	50	50	60	70

The formula for computation of the correlation coefficient between two variables X and Y scores using mean deviation is stated as:

r = \\frac{\\mathbf{\\Sigma }\\left(\\mathbf{X}-\\overline{\\mathbf{X}}\\right)\\left(\\mathbf{Y}-\\overline{\\mathbf{Y}}\\right)}{\\sqrt[]{\\mathbf{\\Sigma }\\left(\\mathbf{X}-\\overline{\\mathbf{X}})^{2}\\mathbf{\\Sigma }\\left(\\mathbf{Y}-\\overline{\\mathbf{Y}})^{2}}} =\\frac{\\mathbf{\\Sigma }\\mathbf{X}\\mathbf{Y}}

{\\sqrt[]{(\\mathbf{\\Sigma }\\boldsymbol{X}^{2})(\\mathbf{\\Sigma }\\boldsymbol{Y}^{2})}}

Where *r* = index of correlation coefficient

x = Deviation of each (x) score from their mean

y = Deviation of each (y) score for the mean

Σ = sum of.

Let us use the data in Table 9.1 to obtain the values following the step by step using mean deviation methods.

Step 1: Write down the scores in column A and B

Sep 2: Sum up the two scores X and Y and compute the mean values for the scores.

Step 3: Record the mean deviation for X scores in column C and that of Y scores in column D

Step 4: Square each deviation scores (X) and (Y) and record them in E and F column respectively.

Step 5: Find the product of x by its corresponding y to obtain xy scores. Write down the results in G.

Step 6: Substitute In the formula to obtain 'r'

S/N	A (X)	B (Y)	C \\left(\\mathbf{X}- \\overline{\\mathbf{X}}) X	D (\\mathbf{Y}- \\overline{\\mathbf{Y}}) Y	E (x^2)	F (y^2)	
1	11	25	–4.5	–19.2	20.25	368.64	
2	12	40	–3.5	–4.2	12.25	17.64	
3	13	45	–2.5	0.8	6.25	0.64	2.(
4	13	20	–2.5	–24.2	6.25	585.64	
5	14	35	–1.5	–9.2	2.25	84.64	
6	15	30	–0.5	–14.2	0.25	201.64	
7	16	40	0.5	–4.2	0.25	17.64	
8	16	45	0.5	0.8	0.25	0.64	
9	17	50	1.5	5.8	2.25	33.64	
10	18	50	2.5	5.8	6.25	33.64	
11	18	60	2.5	15.8	6.25	249.64	
12	19	70	3.5	25.8	12.25	665.64	
13	19	65	3.5	20.8	12.25	432.64	
	201	575	\\overline{\\mathbf{X}} = 15.5	\\overline{\\mathbf{Y}} = 44.2	87.25	2692.32	404.6

Substituting the formula

$$\mathrm{r}=\frac{404.6}{\sqrt{87.25\times 2692.32}}=\frac{404.6}{\sqrt{234904.92}}=\frac{404.6}{484.67}=0.84$$

Let us use the same data In Table 9.1 to calculate the Pearson (r) using the raw score method.

$$r=\frac{\mathbf{N}\mathbf{\Sigma}\mathbf{X}\mathbf{Y}-\left(\mathbf{\Sigma}\mathbf{X}\right)\left(\mathbf{\Sigma}\mathbf{Y}\right)}{\sqrt{\left(\mathbf{N}\mathbf{\Sigma}\boldsymbol{X}^{2}-(\mathbf{\Sigma}\boldsymbol{X})^{2}\right)\left(\mathbf{N}\mathbf{\Sigma}\boldsymbol{Y}^{2}-(\mathbf{\Sigma}\boldsymbol{Y})^{2}\right)}}$$

Where

X and Y = Scores of variables

N = The number of scores.

The following steps can be adopted for calculating Pearson using the raw score method.

Step 1: Enumerate X and Y scores in column A and B

Step 2: Find the sum of X and Y scores to obtain ΣX and ΣY

Step 3: Square X and Y scores to have X^2 and Y^2 and write in the C and D column

Step 4: Sum the square X^2 and Y^2 to have ΣX^2 and ΣY^2

Step 5: Multiply the X and Y scores to obtain XY and write the result in column E

Step 6: Obtain Σ XY in the formula

Step 7: Substitute the values in the stated formula to obtain ‘r’.

Table 9.2: Required Data to compute Pearson ‘r” using Raw scores method

S/N	A (X)	B (Y)	C $\mathbf{X}^{2}$	D $\mathbf{Y}^{2}$	E XY
1	11	25	121	625	275
2	12	40	144	1600	480
3	13	45	169	2025	585
4	13	20	169	400	260
5	14	35	196	1225	490
6	15	30	225	900	450
7	16	40	256	1600	640
8	16	45	256	2025	720
9	17	50	289	2500	850
10	18	50	324	2500	900
11	18	60	324	3600	1080
12	19	70	361	4900	1330

TOTAL 201 575 9295 3125 28125

$201^2 = 40401$ $575^2 = 330625$

$$\mathbf{r}=\frac{\mathbf{N}\mathbf{\Sigma}\mathbf{X}\mathbf{Y}-\left(\mathbf{\Sigma}\mathbf{X}\right)\left(\mathbf{\Sigma}\mathbf{Y}\right)}{\sqrt{\left(\mathbf{N}\mathbf{\Sigma}\mathbf{X}^{2}-(\mathbf{\Sigma}\mathbf{X})^{2}\right)\left(\mathbf{N}\mathbf{\Sigma}\mathbf{Y}^{2}-(\mathbf{\Sigma}\mathbf{Y})^{2}\right)}}$$

$$=\frac{13\times 9295-\left(201\right)\left(575\right)}{\sqrt{\left(13\times 31925-(201)^{2}\right)\left(13\times 28125-(575)^{2}\right)}}=\frac{120835 - 11572}{\sqrt{(41535-40401)(365625 - 330625)}}=\frac{5263}{\sqrt{(1134)(35000)}}=\frac{5263}{\sqrt{(39690000)}}=\frac{5263}{6300}=0.84$$

Since the r value is the same, the researcher has the liberty to use any of formula to calculate Pearson ‘r for interval or ratio data obtained.

9.5 Computation of Spearman Rank Order Method

Another method of calculating correlation coefficient is Spearman rank order correlation developed by Spearman and Brown. It is known as Spearman rho, it is used by ranking each score in variable through the same direction with respect to magnitude, getting the compare the rankings, then square the difference. Apply the formula below:

$$\text{Rho} = 1- \frac{6\sum \mathbf{D}^{2}}{\mathbf{N}(\mathbf{N} - 1)}$$

Where *rho* = Spearman rank-order correlation coefficient

d = Difference between ranks

N = Number of ranks

Let us illustrate the calculation of Spearman rho (rs) by using the data obtained by a Researcher who desires to investigate the relationship between student’s test-anxiety and academic achievement in Geometry.

Table 9.3: Test-anxiety and Academic Achievement Score of Five Students

Test-Anxiety (X)	50	40	60	30	20
Academic Achievement (Y)	80	70	50	60	40

In solving the above example using Table 9.3, the following procedures should be adopted:

Step 1: List out the scores X and Y

Step 2. Rank the scores X and Y from the highest to the lowest. That is to have R_x and R_y

Step 3: Determine the square of each difference (d) Add the d^2 to obtain Σd^2

Step 5: Substitute in the formula and calculate the rho value.

(X)	(Y)	Rx	Ry	D	D^2
50	80	2	1	1	1
40	70	3	2	1	1

60	50	1	4	–3	9
30	60	4	3	1	1
20	40	5	5	0	0

Rho = 1- \\frac{6\\sum \\boldsymbol{D}^{2}}{\\mathbf{N}(\\mathbf{N}\\hbox{--}1)} = 1 - \\frac{5\\mathrm{x}12.00}{5(5-1)}

= 1\\hbox{--}\\frac{60}{5\\left(25\\hbox{--}1\\right)}

= 1 - \\frac{60}{5(24)}

= 1 - \\frac{60}{120}

= 1—0.5

= 0.5

By the result obtained Rho = 0.5, there is positive correlation between test-anxiety and academic achievement.

The type and strength of correlations between the two or more variables under study are always determined by a very excellent research study. Correlation refers to the strength of the link that exists when comparing variables, and the correlation coefficient is the statistical index used to measure the relationship.

Student Activity

1. Using mean deviation and raw score method to compute the correlation coefficient of the data obtained by researcher and comment on the result.

S/N	1	2	3	4	5	6	7	8	9	10	11	12
X	12	17	62	27	53	33	26	52	14	47	49	11
Y	05	07	57	13	46	31	23	27	13	32	43	03

1. Motivational Level and Academic Performance Scores of Secondary Students

Students	1	2	3	4	5	6	7	8	9	10
Motivational Level Score (X)	5	8	10	12	13	8	20	15	17	18
Academic Performance (Y)	40	45	50	55	60	50	65	55	62	60

Calculate the correlation coefficient using any practical correlation approach.

1. Using the data below, to calculate the Spearman Rank order correlation(rho).

Students	A	B	C	D	E	F	G	H	I	J

X Score	51	44	80	45	71	70	32	65	19	67
Y Score	49	41	75	21	64	45	31	50	11	61

1. Below are scores in multiple choice test in each of a unit in Mathematics and Physics. By 10 students: scores are X and Y respectively

S/N	1	2	3	4	5	6	7	8	9	10
Mathematics (X)	5	8	10	6	17	19	12	9	12	8
Physics (Y)	10	8	7	9	9	6	6	8	6	9

1. Compute the Pearson r between X and Y using mean deviation method.
2. Calculate the Pearson ‘r’ between X and Y using raw scores approach.
3. Compute the Spearman rho based on the data provided above.

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CHAPTER TEN

HYPOTHESES TESTING

10.1Introduction

You will learn what a hypothesis is and the many kinds of hypotheses in this chapter. There are certain fundamental ideas in testing hypotheses that you will learn, like level of significance, degree of freedom, type I and type ii error, etc. You should consequently be able to do statistical analysis in your research investigations after reading this chapter.

10.2Objectives

At the end of this chapter, you should be able to:

1. define hypothesis and its types.
2. describe some basic concepts in testing hypothesis
3. enumerate the procedure for testing hypothesis and explain each step involved.

10.3Hypothesis

Hypothesis is a term referred to be an assertion subject to verification or an assumption used as a basis of action. Hypotheses are, therefore, assertions or an assumption made and not established facts. It is a sign of how seriously researchers take the solutions to their stated research challenges. When a hypothesis is tested, the findings can produce new information or validate already-known knowledge. A theory is supported by a hypothesis if it can be tested, validated, and verified to be true. In other words, the process of evaluating hypotheses in research involves testing and confirming theories, which advances knowledge.

10.3.1Types of Hypotheses

Hypotheses are stated into forms: null hypothesis and alternative hypothesis. A null hypothesis denoted by (H_0) is a hypothesis that is stated in the form no difference or no relationship between variables under analysis.

An alternative hypothesis (H_a) stated in the form, a difference or relationship exists between two populations or between two parameters of two populations. It is hypothesis that is retain when the null hypothesis is rejected. The alternative hypothesis sometimes refers to be directional if it shows the direction of difference or relationship, while non-directional does not show the direction of difference or relationship between variables under analysis.

10.4Significance Level Selection

Selection of the level of significance is sometimes known as alpha level (α) or significance level. In psychological and educational research, the alpha level is expressed at.05 and.01 levels of significance. When we set our alpha (significance) level at.01, we are indicating that if the research is repeated 100 times, 99 out of 100 times, the same result will occur, and 1 out of 100 times, the result may differ owing to risk. However, if it is set at.05, it indicates that if the research is repeated 100 times, you can be certain that 95 of the results would be accurate, while 5 of the 100 results might differ owing to chance variables or risk factors.

Anytime, a Researcher is stating a null hypothesis, it is very important to give it a chance of rejection. On this basis, the data collected by the Researcher for analysis is at a point of rejection, you must specify the level at which to bear the risk. This risk is stated in probability level for being true or false. It can also be considered as the amount of error involved in a given statistical decision about the null hypothesis. This therefore indicates that selection of a specific level of significance, precisely the amount of error risk involved in the decision.

10.5Degrees of Freedom

The term degree of freedom means the number of independent observations or values in a sample that are free to vary when computing a test statistically. It is the probability that the test will lead to a decision to reject H_0 when H_0 is indeed false. Take for instance, you have a sample 30 cases, the degree of freedom is 30—1 = 29. This means that 29 of the 30 observations are chance to vary and hence (n—1) degree of freedom. In another way, you ask your Students in class that add three to 5 to get 25. In this instance, 5 is a fixed number, while other numbers are flexible. N is the total number of observations or chances, and 1 is the fixed variable, hence the degree of freedom is N—1. As you will see later, different statistical tests use different methods to determine the degree of freedom.

10.6Type I and II Errors

When the calculations are made correctly, the Researcher will make correct decision about null hypothesis, but still can commit two possible errors in decision making about null hypothesis. However, a Type I mistake occurs when a correct null hypothesis is rejected when it should have been preserved. The symbol for this is alpha (α).

In the opposite direction, if you accept the null hypothesis rather than reject it when it is wrong, you are incorrect. Type II mistake is the acceptance of a false null hypothesis when it ought to have been rejected. Beta serves to indicate the mistake (\\beta).

Note that the Researcher is trying to minimize type I error and type II error by stating the level of significance at 0.1 to 0.5. if the probability of accepting a false (H_0) is

'\\beta ' then the probability of rejecting a false H_0 is 1—\\beta . this is known as the power of the statistical test.

10.7One-tailed and Two-tailed Tests

Because the null hypothesis does not reveal the direction of the difference, the test is always two-tailed. For instance, a null hypothesis claims that there is no connection between academic success and motivational level.

One-tailed test is used when the hypothesis is presented and shows the direction of the association. For instance, academic success and motivational level are significantly correlated.

10.8Testing of Hypothesis

Testing of hypothesis is kind of decision-making process in a research work. It is an idea to verify the proposed null hypothesis of its benefit of doubt. This opportunity for rejecting null hypothesis can also be viewed as the strategy for choosing between hypotheses and it involve severed steps. These steps are as followed:

Step 1: Stated statement of hypothesis.

Step 2: Selecting the significance (alpha) level (α)

Step 3: Choosing relevant test statistic and applied it to obtain values from sample data

Step 4: Determining the critical region (i.e., rejection and acceptance region)

Step 5: Making valid decision based on the result (i.e., statistical decision)

Step 6: Conclusion.

10.8.1Statement of Hypothesis

Hypothesis is stated to guide the research activities, and the Researcher, therefore there is the need to verify the statement through the process of hypothesis testing.

10.8.2Selecting of Alpha Level

A researcher's study effort must include testing the hypothesis. The alpha level and degree of freedom allow the researcher to make an informed judgment on whether to keep or discard the hypothesis. Additionally, it helps to reduce type I and type II errors in research studies.

10.8.3Statistical Decision

A Researcher, after computing all values for the test statistically, it is now left to compare calculated value with critical values in the table, that enable to accept or reject the hypothesis. Since you are aware of level of significance and concept of degree of freedom in our previous units.

10.8.4Drawing Conclusion

After statistical decision, the next action is to draw conclusion. The conclusion is drawn based on the results obtained.

10.9Choices of Appropriate Statistical Tools

Whenever a researcher wants to describe the general nature and characteristics of a given sample, the researcher uses descriptive statistical, such as percentage, frequency distribution table, and cumulative frequency and graphical representation of data (bar, frequency polygon, histogram, pie chart etc). Furthermore, the appropriate descriptive measure that uses a score to describe or represent others in the sample is measure of central tendency (i.e. mean, mode and median). While the appropriate descriptive measure for relative position is sigma score, percentile, quartiles, rank and standard scores (Z or T- scores). In descriptive statistics the generalization is limited to the particular group of the research, i.e, the generalization of the descriptive statistical measures is limited to the sample of the research. This implies, no conclusion could be extended beyond the sample of the study. In addition, it also provides valuable and useful information about the nature of the sample only. In the other hand, inferential statistics can be use if the data obtained is either parametric or non-parametric and the followings statistical tools that can be used for testing hypothesis are:

1. *Correlations or Relationships:* The researcher that wants to find out correlations or relationships between variables. If the basic assumptions for parametric is meant. The researcher makes use of Pearson Product Moment Correlation (P. P.M.C) statistics. But a situation if conditions of basic assumptions of parametric are not meant, the researcher makes use of Spearman Rank correlation (Rh_o) or chi-square (x^2)
2. *Differences:* The researcher that wishes to investigate the differences between variables the appropriate statistics tools are discuss below:
3. The researcher makes use of t-test if it is parametric data and has single sample with population mean.
4. The researcher makes use of t-test statistic if the data collected is parametric nature with two samples.
5. The researcher makes use of Analysis of Variance (ANOVA), if the data obtained is parametric with three or more samples.
6. The researcher makes use of krustall-wallis test if it is non-parametric.

7. *Effects/Influence:* The appropriate statistical tools for finding out interactive effects /influence are:
8. The researcher employs the use of Analysis of Co-Variance (that is per-test Vs post-test), Two-way ANOVA (i.e. interactive between two variables), if it is parametric (interactive effects).
9. The researchers equally use Two-way ANOVA (i.e. interaction between two variables), if it is parametric (influence).

Choice of Appropriate Statistical Tool on the Basis of Measurement Scales

1. *Nominal scale:* It is measurement which does not bear any magnitude relationship to one another, the appropriate statistical tools for hypotheses testing under this data are Signed test, Chi-square, Phi-coefficient etc.
2. *Ordinal scale:* This measurement at this level which bears only magnitude and possess all the attributes of nominal scales, the appropriate statistical tools used for analysing are Spearmen rank correlation, Rank sum, U-test, Krustal-wallis.
3. *Interval scale:* This scale has all the attributes of nominal, ordinal and equal interval between measurements of units. The researcher makes use of T-test, ANOVA, Correlation analysis etc.
4. *Ratio scale:* This scale has properly of absolute zero and has all attributes of nominal and interval. To test hypotheses for this scale measurement the researcher makes use of Z-scale, Measures of Central Tendency, ANOVA, ANCOVA.

Student Activity

1. Distinguish between the following pairs of concepts.
2. Level of significance and degree of freedom
3. Type I error and Type II error
4. One-tailed and two-tailed test.
5. Explain the following terms
6. $P < 0.01$
7. Alternative hypothesis
8. Null hypothesis
9. Power of statistical test.
10. Briefly describe the procedure for testing of hypothesis in a research study.

References

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CHAPTER ELEVEN

INFERENTIAL STATISTICS I

11.1 Introduction

Inferential Statistics describes population depending on the sample's behavior. It also involves figuring out if the outcomes based on the sample or samples match the outcomes that would have been achieved for the complete population. Inferential statistics are used to predict population features from a randomly chosen sample as well as to estimate population parameter using sample data. They are used to test formulated hypothesis to draw a valid conclusion from research studies. This chapter deals with types of inferential statistics, computation of t-test, Z-test, correlation analysis, ANOVA or F-Ratio.

11.2 Objectives

At the end of this chapter, you should be able to:

- learn the types of inferential statistics
- describe situations where the use of z-test, t-test and f-test are applicable.
- compute z-test with relevant examples
- compute t-test with relevant examples
- compute f-test with relevant examples
- use t-test for hypothesis testing for difference between population and sample means
- use t-test for hypothesis testing for difference between correlation coefficients.

11.3 Types of Inferential Statistics

Inferential statistics are categorized into two: Parameter statistics and non-parameter statistics. The two statistics are useful when testing hypothesis in research. However, parametric statistics is more powerful and generally preferred. By more powerful is meant that it requires certain assumptions which must be considered in order to make valid decision. The followings are three very important assumptions that are made when applying parametric statistics to test a formulated hypothesis:

The variable measured is normally distributed or at least in the form of distribution must be known.

The data collected must be from interval or ratio scale of measurement

The variable must be independently selected without affecting the selection of any other one.

The researcher must note that any one or more of these assumptions discarded, then non-parametric inferential statistical test should be employed. This chapter will only discuss t-test, Z-test, ANOVA.

11.4 The Z-Test

The Z-statistic is applied to investigate whether two means are significantly different. It is used for testing hypothesis when the sample size is equal or greater than 30 (≥ 30). When the population parameters μ and σ , are well-defined for a population. The Z-test formula is given as:

$$Z = \frac{\overline{\mathbf{x}}_{1}-\overline{\mathbf{x}}_{2}}{\mathbf{S}\mathbf{D}}$$

Where $\overline{\mathbf{x}}$ = Calculate mean score

SDx = Standard error of difference between means

$$SDx = \sqrt[]{\frac{{\mathbf{S}_{1}}^{2}}{\mathbf{n}_{1}}+\frac{{\mathbf{S}_{2}}^{2}}{\mathbf{n}_{2}}}$$

$$\therefore Z = \frac{\overline{\mathbf{x}}_{1}-\overline{\mathbf{x}}_{2}}{\sqrt[]{\frac{{\mathbf{S}_{1}}^{2}}{\mathbf{n}_{1}}+\frac{{\mathbf{S}_{2}}^{2}}{\mathbf{n}_{2}}}}$$

Where $\overline{\mathbf{x}}_{1}$ = Mean of group 1

$\overline{\mathbf{x}}_{2}$ = Mean of group 2

SDx = Standard error of difference between means

Now, let us demonstrate the use of Z-test in hypothesis testing with an example given below:

Analyze the given data representing set of scores obtained by five students from Mathematics and Chemistry test.

Table 11.1: Sets of Scores obtained in Mathematics and Chemistry Test

Students	Maths Scores X1	Chem Scores X2
1	4	4
2	5	4
3	6	5
4	7	3
5	8	4

Based on the data provided in Table 11.1, we shall solve this problem by going through the process of hypothesis testing.

Step 1: Statement of Hypotheses

H0: $\mathbf{\mu} = x$

H1: $\mathbf{\mu} \neq x$

Step 2: Determine the level of significance

Assuming $\alpha = 0.05$ is selected

Step 3: Calculate the test statistics by applying the formula provided as:

Z = \\frac{\\overline{\\mathbf{x}}_{1}-\\overline{\\mathbf{x}}_{2}}{\\mathbf{S}\\mathbf{D}}, SDx = \\sqrt[]{\\frac{{\\mathbf{S}_{1}}^{2}}{\\mathbf{n}_{1}}+\\frac{{\\mathbf{S}_{2}}^{2}}{\\mathbf{n}_{2}}} given Z = \\frac{\\overline{\\mathbf{x}}_{1}-\\overline{\\mathbf{x}}_{2}}{\\sqrt[]{\\frac{{\\mathbf{S}_{1}}^{2}}{\\mathbf{n}_{1}}+\\frac{{\\mathbf{S}_{2}}^{2}}{\\mathbf{n}_{2}}}}

x1

\\mathbf{x}_{1}-\\overline{\\mathbf{x}}

(\\mathbf{x}_{1}-\\overline{\\mathbf{x}})2

\\mathbf{x}_{2}

\\mathbf{x}_{2}-\\overline{\\mathbf{x}}

(\\mathbf{x}_{2}-\\overline{\\mathbf{x}})2

4

2

4

4

0

0

5

1

1

4

0

0

6

0

0

5

1

1

7

1

1

3

–1

1

8

2

4

4

0

0

\\mathbf{\\Sigma }\\mathbf{x}_{1}=30

\\mathbf{\\Sigma }\\left(\\mathbf{x}_{1}-\\overline{\\mathbf{x}}\\right)2

ΣX2 = 20

Σ(X2-\\overline{\\mathbf{x}}) = 2

\\overline{\\mathrm{x}}1 = \\frac{30}{5}=6 ΣX2 - \\frac{20}{5}=4

SD = \\sqrt[]{\\frac{10}{5}} = \\sqrt[]{\\frac{2}{5}}

= \\sqrt[]{2} = \\sqrt[]{0.4}

S1 = 1.41 S2 = 0.63

Now substitute in z = \\frac{\\overline{\\mathbf{x}}_{1}-\\overline{\\mathbf{x}}_{2}}{\\sqrt[]{\\frac{{\\mathbf{S}_{1}}^{2}}{\\mathbf{n}_{1}}+\\frac{{\\mathbf{S}_{2}}^{2}}{\\mathbf{n}_{2}}}}

= \\frac{6\\hbox{--}4}{\\sqrt[]{\\frac{\\left(1.41\\right)^{2}}{5}+\\frac{\\left(0.63\\right)^{2}}{5}}} = \\frac{2}{\\sqrt[]{\\frac{1.99}{5}+\\frac{0.40}{5}}} = \\frac{2}{\\sqrt[]{0.40+0.8}}=\\frac{2}{\\sqrt[]{0.48}}=\\frac{2}{0.69} = 2.90

Step 4: Determine the critical region. At p = 0.05 level of significance, the critical or table value of z = ±1.96 and calculated Z-value is 2.90.

Step 5: Decision. By the available records the calculated Z-value is 2.90 greater than the. Z table value 1.96. Therefore, the null hypothesis is rejected.

Step 6: Conclusion. By the result obtained, it is concluded that there is significant difference between the two mean scores under statistical analysis.

11.5The T-Test

Student t-test was the name given to the t-test statistic. Williams Gosset created it as an inferential statistic in 1908. The t-test statistic offers a number of methods for testing hypotheses; however, we will just cover the following in this unit:

Use the t-test to determine if two independent samples' mean scores differ significantly.

T-test for a non-independent significant difference between two mean scores samples.

Thus, getting to computations of t-test, there are some conditions to be satisfied before using the t-test. These are as follows:

A comparison of two groups is required.

The sample that is chosen must have a normal distribution.

Population variance is homogeneous.

The samples are independently or at random chosen from the population.

The requirements for the variable values must hold.

Both big and small samples can be utilized for the t-test, although the sample size cannot be fewer than ten.

11.5.1Computation of T-Test for Difference Between Two Independent Samples

When two independent samples' mean scores are given, it is possible to assess whether there is a significant difference between them as:

$$t = \frac{\overline{\mathbf{x}}_{1}-\overline{\mathbf{x}}_{2}}{\sqrt{\frac{\mathbf{S}_{1}^{2}}{\mathbf{n}_{1}}+\frac{\mathbf{S}_{2}^{2}}{\mathbf{n}_{2}}}}$$

where $\overline{\mathbf{x}}_{1}$ = mean scores of sample group 1

$\overline{\mathbf{x}}_{2}$ = Mean scores of samples group 2

$\mathbf{S}_{\mathbf{x}}^{2}$ = Variance

n = Sample size.

Let us now, demonstrates the calculation of t-test with data provided in Table 11.2 assumption, that the following conditions to use t-test are satisfied.

The distribution of the value in both samples normal.

Data collected are interval measurement scale.

Sample is randomly selected.

Sample variances are homogeneous.

Table 11.2 A set of pre-service Teachers took post-test for two samples randomly selected

S/N	Group A
1	04
2	16
3	16
4	15
5	14
6	15
7	10
8	17
9	17
10	18
11	18
12	20

Group B

10

12

18

13

6

16

10

15

05

19

09

11

Are the results significant different, or not?

Since the Researcher is interested to determine whether significant difference exists between the mean value of Group A and B. Let us illustrate the computation for t-test or unrelated or independent samples. We shall adopt the steps for hypothesis testing in solving the problem in Table 11.2

Step 1: Statement of Hypotheses

H0: $\mathbf{\mu}_{\mathbf{A}}= \mathbf{\mu}_{\mathbf{B}}$

H1: $\mathbf{\mu}_{\mathbf{A}}\neq \mathbf{\mu}_{\mathbf{B}}$

Step 2: Level of significance is at 0.05

Step 3: Calculate the t-test by applying the formula:

$$t = \frac{\overline{\mathbf{x}}_{1}-\overline{\mathbf{x}}_{2}}{\sqrt{\frac{{\mathbf{S}_{1}}^{2}}{\mathbf{n}_{1}}+\frac{{\mathbf{S}_{2}}^{2}}{\mathbf{n}_{2}}}}$$

Firstly, let us label each group as Group A = X1 and Group B = X2, we then calculate mean scores.

$\mathbf{x}_{1}$

$\mathbf{x}_{1}-\overline{\mathbf{x}}$

$(\mathbf{x}_{1}-\overline{\mathbf{x}})2$

$\mathbf{x}_{2}$

$\mathbf{x}_{2}-\overline{\mathbf{x}}$

$(\mathbf{x}_{2}-\overline{\mathbf{x}})2$

04

–11

121

10

–2

4

16

1

1

12

0

0

16

1

1

18

6

36

15

0

0

13

1

1

14

–1

1

6

–6

36

15

0

0

16

4

16

10

–5

25

15

3

9

17

2

4

05

–7

49

17

2

4

19

7

49

18

3

9

09

–3

9

18

3

9

11

–3

1

20

5

25

$\mathbf{\Sigma}\mathbf{x}_{1}=360$

$\mathbf{\Sigma}\left(\mathbf{x}_{1}-\overline{\mathbf{x}}\right)2$

$\mathbf{\Sigma}\mathbf{x}_{2} = 144$

$\mathbf{\Sigma}(\mathbf{x}_{2}-\overline{\mathbf{x}}) = 214$

$\overline{\mathrm{x}}_{1}=\frac{360}{12}=15$ $S2=\frac{200}{12}=16.67$

$\overline{\mathrm{x}}_{2}=\frac{144}{12} = 12$ $S2 = \frac{214}{12}=17.83$

$S = \sqrt{16.67}=4.08$ $S = \sqrt{17.83}=4.22$

Therefore, $t = \frac{15.00\hbox{--}12.00}{\sqrt{\frac{4.08}{12}+\frac{4.22}{12}}}= \frac{3}{\sqrt{\frac{8.3}{12}}} = \frac{3}{\sqrt{0.69}} = \frac{3}{0.83} = 3.75$

Step 4: Critical region is determined by $\alpha = 0.05$ level of significance and degree of freedom (αf) = 12 + 12—2 = 22 looking for t-value or table value, which gives 2.074

Step 5: Decision. Thus, the t-calculated is 3.75 and critical value is 2.074, since the t-calculated is greater than the critical value, the null hypothesis is rejected.

Step 6: Conclusion, based on the result obtained, we shall conclude that there is significant difference between the two groups.

11.5.2Computation of T-Test for Non-Independent Samples

Researchers occasionally get into situations where they must compare student performance across two unrelated or closely related courses. When this occurs, the t-test for non-independent samples is used to determine if the mean scores of two matched or non-independent samples differ significantly from one another. The calculation formula is as follows:

$$\mathbf{t}=\frac{\sum \mathbf{d}}{\sqrt[]{\frac{\mathbf{N}\sum \mathbf{d}^{2}-(\sum \mathbf{d})^{2}}{\mathbf{N}-1}}}$$

Where d = difference between each material samples

Σd = Addition of the differences between the matched samples

d2 = Square of the difference between each matched sample.

N = total matched samples

N—1 = number of degree freedom

For example, research administered in both Mathematics and Chemistry with scores as follows:

Table 11.3: Data Obtained from Two Subjects

S/N

1

2

3

4

5

6

7

8

9

10

Mathematics

10

25

50

37

80

23

48

63

40

35

Chemistry

29

48

46

17

30

45

19

48

42

50

Is the result significantly different?

Let us solve the problem in Table 11.3 using the procedure for testing hypothesis.

Step 1: Statement of hypotheses

H0: $\mathbf{\mu}_{1} = \mathbf{\mu}_{2}$

H1: $\mathbf{\mu}_{1} \neq \mathbf{\mu}_{2}$

Step 2: Selection of level of significance. $\alpha = 0.05$ two-tailed test

Step 3: Calculate the t-test by going through the data provided in table 11.3

Students	Mathematics $\mathbf{x}_{1}$	Chemistry $\mathbf{x}_{2}$	D $\mathbf{x}_{1} - \mathbf{x}_{2}$	D2
1	10	29	19	361
2	25	48	–23	529
3	50	46	4	16
4	37	17	20	400
5	80			

30

50

2500

6

23

45

–22

484

7

48

19

29

841

8

63

48

15

225

9

40

42

–2

04

10

35

50

–15

225

Σ

104

5585

Substitute in the given formula as: $\mathbf{t}=\frac{\sum \mathbf{d}}{\sqrt[]{\frac{\mathbf{N}\sum \mathbf{d}^{2}-(\sum \mathbf{d})^{2}}{\mathbf{N}-1}}}$

$\Sigma d = 104$, $d2 = 5,585$, $(\Sigma d)2 = (104)2 = 10,816$

$\mathbf{t}=\frac{\sum \mathbf{d}}{\sqrt[]{\frac{\mathbf{N}\sum \mathbf{d}^{2}-(\sum \mathbf{d})^{2}}{\mathbf{N}-1}}}=\frac{104}{\sqrt[]{\frac{10\times 5585-(104)^{2}}{10-1}}}=\frac{104}{\sqrt[]{\frac{55850\hbox{--}10816}{9}}}=\frac{104}{\sqrt[]{\frac{45034}{9}}}= \frac{104}{\sqrt[]{5003.78}} = \frac{104}{70.74} = 1.47$

Step 4: Now critical region is determining as $\alpha = 0.05$ with df = n—1 = 10—1 = 9.

Looking at t-critical table and search for t-value with df = 9 at $\alpha = 0.5$. which shows that the t-value is 1.83

Step 5: Decision; Since our calculated t (1.47) is less than the critical t (1.83) then the H0 is retained.

Step 6: Conclusion, since t cal < t tab, we RETAIN that there is no significant difference in the results or the results are not significant difference.

11.5.3T-test for Difference Between Population and Sample Means

When a Researcher want to compare a population and sample means, Researcher make use of this formula:

$t=\frac{\overline{x}-\mathrm{\mu}}{\sfrac{S}{\sqrt[]{n\hbox{--}1}}}$

Where $\overline{x}$=sample mean

$\mathrm{\mu}$= Population mean

s= Standard deviation

n= Number.

For example, A Researcher conducted a study and obtained the mean achievement score of all SS II students in senior secondary schools, in Bida Local Government Area in Post-test as 25.50%. Another Researcher carryout a study to verify that result and used 15 SS II Students sampled out in that study area. He then gave treatment on for areas of mathematics, for six weeks. At the end of treatment, the Researcher administered the mathematics achievement test (MAT) and obtained the following results: means are 30.10, 7.5 standard deviation.

Based on the data obtained, we shall solve the by going through the process of hypothesis testing.

Step 1: Statement of Hypotheses

H0$\colon \mathrm{\mu}=\overline{x}$

H1$\colon \mathrm{\mu}\neq \overline{x}$

Step 2: Determine the level of significance

Assuming $\alpha =0.05$ is selected.

Step 3: Compute the test statistics by using the formula:

$=\frac{\overline{x}-\mathrm{\mu}}{\sfrac{S}{\sqrt[]{n\hbox{--}1}}}$

Where $\overline{x}=30.10,\mathrm{\mu}=25.50,s=7.5,n=15$.

$\therefore$ $t=\frac{30.10\hbox{--}25.50}{\sfrac{7.5}{\sqrt[]{15\hbox{--}1}}}=\frac{4.6}{\sfrac{7.5}{\sqrt[]{14}}}=\frac{4.5}{\sfrac{7.5}{3.7}}=\frac{4.6}{2.03}=2.27$

Step 4: determine the critical region, at $\alpha =0.05$

Level of significance. Now that t-calculated = 2.27, df = 15 –1 = 14, alpha level =0.05 then the t-critical = 2.13

Step 5: Decision on rule, if calculated value is greater than the critical value then the null hypothesis is rejected. But if the t-calculated value is less than the critical value, the null hypothesis is retained.

From the result obtained, t-calculated is greater than the t-critical i.e. 2.27, 2.13. We therefore rejected null hypothesis.

Step 6: Conclusion: we concluded that there is a significant difference between the two means.

11.5.4Computation for Difference between Correlation Coefficients

Testing hypothesis about correlations have two approaches: the first one which you are familiar with, is to use the table and find out if the correlation coefficient is significant, while the second way is by using the correlation coefficient directly from the table then you can subject it to a t-test. Using below formula:

t=\\frac{\\sqrt[]{1-r^{2}}}{n\\hbox{--}2}\\mathrm{o}\\mathrm{r}t=\\frac{r\\sqrt[]{n\\hbox{--}2}}{1-r^{2}}

For instance, a Lecturer want to investigate whether students' scores in MAT 201 have any significant relationship with their scores in MAT 301. He then used applied Pearson Product Moment Correlation. He obtained results as r = 0.70, N = 40. Find out whether there is significant relation.

Let us use the process of hypothesis testify to solve the above problem.

Step 1: Propose a null hypothesis

There is no significance relationship between the students' scores in both MAT 201 and MAT 301.

Step 2: Select the level of significance. At \\alpha =0.05 level of significance is assumed.

Step 3: Calculate the t-test using the formula:

\\frac{r\\sqrt[]{n\\hbox{--}1}}{1-r^{2}}

Giving that r = 0.70, n = 40 substitute with formula as

= \\frac{0.70\\sqrt[]{40\\hbox{--}2}}{\\sqrt[]{1\\hbox{--}0.70^{2}}}=\\frac{0.70\\sqrt[]{38}}{\\sqrt[]{1\\hbox{--}0.70^{2}}}=\\frac{0.070\\times 6.16}{\\sqrt[]{1\\hbox{--}0.49}}=\\frac{4.312}{\\sqrt[]{0.51}} =\\frac{4.312}{0.714} =6.04

Step 4: Determine the critical region as at\\alpha =0.05 level of significant and t-calculated is 6.04.with df = 40—1 = 39, then critical value is at 2.021

Step 5: Decision: now decision is taken since t-calculated greater than t-critical i.e., 6.04 > 2.021, the null hypothesis is rejected.

Step 6: Conclusion: Based on the results obtained we conclude that there is significant relationship between MAT 201 and MAT 301.

Student Activity

Differentiate between parametric and non-parametric test

State three conditions for using parametric test.

What is t-test?

Differentiate between t-test and z-test

What is z-test?

What are those conditions to be looked into, before choosing t-test?

Analyse the given data representing the set of scores from day and boarding schools. Use the t-test to determine significance difference or not.

Day (D)

26

15

8

44

26

13

38

24

13

29

Boarding (B)

20

4

9

36

20

3

25

10

6

14

The Researcher obtained the following scores for the experimental and control groups.

Experimental Group

30

64

47

38

59

81

44

Control Group

20

24

31

18

57

26

10

Find out whether these sets of scores are significantly different or not using t-test for non-independent samples.

Using t-test for independent samples with data provided

Group 1

10

11

13

14

15

16

17

18

19

20

Group 2

9

10

12

13

13

13

14

14

15

16

Are the results significant different or not?

Suppose the Researcher obtains sets of scores

Score (x1)

3

4

5

6

2

7

8

9

10

11

Score (x2)

2

3

3

3

4

4

5

5

6

6

Compute using z-test find out whether the set of scores are significantly different.

The Researcher conducted studies and obtained the following data provide below:

Population

Mean

Sample

Mean

Sample

Size

Standard

Deviation

1st Researcher

55%

59.85

25

8.50

2nd Researcher

65%

70.15

45

11.50

3rd Researcher

58%

65.01

40

14.50

Find out whether performance significant different?

Using \\alpha =0.5 level of significant.

In research conducted, it was found that the correlation coefficient of two variable was 0.85 and the number of the respondents, was 50. Propose a null hypothesis and test using \\alpha at 0.05 levels.

11.6Analysis of Variance (F-test)

R. A. Fisher created the acronym ANOVA, or Analysis of Variance, in 1923. Since then, researchers have utilized it frequently and broadly. It is a parametric test that assesses if there is a statistical link between the variables being analyzed by contrasting the mean scores of three or more groups. Whenever a researcher wants to find out if two or more independent samples taken from populations with similar mean scores have significantly different mean scores, they should do so, F-test is the best statistical test to be used. Because ANOVA not only eliminates the differences but also brings out the cause or causes of such significant difference.

The basic rule of ANOVA is comparing the amount of variance 'between the samples' with that of the "within the samples". This comparison is carried out by dividing the variance 'between samples' with the variance of 'within samples' to obtain a ratio known as F-ratio. There are two major types of ANOVA, one-way ANOVA, and two-way ANOVA.

11.6.1Computation of ANOVA (F-test)

The researcher must note that some basic assumptions are considered before applying F-test in any research studies. These assumptions are:

The samples selected from the population should be independent random samples.

The variance in the population should be normal distributed.

The data generated should be interval in nature

Homogeneity of variances is necessary

In carryout the F-test the following steps should be consider:

Step 1: calculate the sum of squares (Σx2) and sum of scores (Σx) for each group in the data provided.

Step 2: calculate the scores for all combined groups into composite group called as the total group variance (Vt), given as.

SStotal (Vt) = Σx2 - $\frac{(\sum \mathbf{X})^{2}}{\mathbf{n}}$

Step 3: Find the difference between the total group variance and the within groups variance known as the between-groups variance (Vt—Vw - Vb)

The formula is given as:

SSbetween (Vb) = ($\sum$ xg)2 - $\frac{(\sum \mathbf{X})^{2}}{\mathbf{n}}$

Where SSbetween = Between sum of square

ng = the number of individual scores in each group

n = the total number of individual scores in all the groups.

$\frac{(\sum \mathbf{X}\mathbf{g})^{2}}{\mathbf{n}}$ = the sum of each group's raw scores squared and divided by ng

$\frac{(\sum \mathbf{X})^{2}}{\mathbf{n}}$ = the sum of all raw sores squared and divided by n

Step 4: The mean value of the variance of each group calculated separately is known as the within groups (Vw). this is done by

SStotal—SSbetween = SSwithin and the formula below is applied:

\\mathbf{x}_{\\mathbf{g}}^{2} - \\frac{(\\sum \\mathbf{X}\\mathbf{g})^{2}}{\\mathbf{n}}

Step 5: At this step, you need to determine all degrees of freedom. The degrees of freedom are given as:

Total degrees of freedom (Dftotal) = n—1

Between groups df(Dfbetween) = k—1

Within groups df(Dfwithin) = n—k

Where n = total number of cases and k = numbers of groups.

Step 6: to compute for the between groups mean square, it is obtaining between sum of squares divided by the between df as:

MSbetween = \\frac{\\mathbf{S}\\mathbf{S}_{\\mathbf{s}}}{\\mathbf{d}\\mathbf{f}_{\\mathbf{g}}}

Similarly, that of within group mean square given by dividing the within sum of squares by within df as: \\frac{\\mathrm{S}\\mathrm{S}\\mathrm{w}}{\\mathrm{D}\\mathrm{f}\\mathrm{w}}

(between-groups variance)

(within-groups variance)

(between-groups variance)

(within-groups variance)

Step 7:

F = \\frac{\\mathbf{M}\\boldsymbol{S}_{\\mathbf{b}\\mathbf{e}\\mathbf{t}\\mathbf{w}\\mathbf{e}\\
{\\mathbf{M}\\boldsymbol{S}_{\\mathbf{w}\\mathbf{i}\\mathbf{t}\\mathbf{h}\\mathbf{i}\\mathb
\\frac{\\mathbf{V}_{\\mathbf{b}}}{\\mathbf{V}_{\\mathbf{w}}}

Let us demonstrate how to calculate F-test using the example below:

Develop an ANOVA table for the following data on the sets of scores of students after treatment using three teaching strategies on three groups of students. Are the strategies have significant differences on the performance of the students at \\alpha = 5% level of significance.

Table 11.4: Data on Set of Scores for the Three Groups

Discovery

2

3

5

7

6

5

4

3

5

Scaffolding

4
3
2
5
7
9
5
2
1
3
Laboratory
5
7
6
7
5
3
2
4
6
2
1

Let us demonstrate the calculation in F-test with the data in Table 11.4 by using the steps for hypothesis testing.

Step 1: Statement of hypotheses

H0: $\overline{\mathbf{x}}_{1}=\overline{\mathbf{x}}_{2}=\overline{\mathbf{x}}_{3}$

H1: $\overline{\mathbf{x}}_{1}\neq \overline{\mathbf{x}}_{2}\neq \overline{\mathbf{x}}_{3}$ (not all the mean scores are equal)

Step 2: Formulate the level of significance as $\alpha =.05$

Step 3: Select the appropriate test statistics for research study. The F-test is appropriate test statistic

F =
\\frac{\\mathbf{B}\\mathbf{e}\\mathbf{t}\\mathbf{w}\\mathbf{e}\\mathbf{e}\\mathbf{n}\\mathb
{\\mathbf{w}\\mathbf{i}\\mathbf{t}\\mathbf{h}\\mathbf{i}\\mathbf{n}\\mathbf{s}\\mathbf{a}\\m
$\frac{\mathbf{V}_{\mathbf{b}}}{\mathbf{V}_{\mathbf{w}}}$

k—1 degrees of freedom for numerator and N—k degrees of freedom for denominator, where K stands for number of treatment and 'n' for total number of observations.

Now composite the table of value to calculate for Σx and Σx2 for each group.

Table 11.4: Data of F-Statistic for Three Group

Group 1

Group 2

Group 3

\\mathbf{x}_{1}

\\mathbf{x}_{1}^{2}

\\mathbf{x}_{2}

\\mathbf{x}_{2}^{2}

\\mathbf{x}_{3}

\\mathbf{x}_{3}^{3}

2

04

4

16

5

25

3

09

3

09

7

49

5

25

2

04

6

36

7

49

5

25

7

49

6

36

7

49

5

25

5

25

9

81

3

09

4

16

5

25

2

04

3

09

2

04

4

16

5

25

1

01

6

36

3

09

2

04

1

01

$\mathbf{n}_{1} = 9$ $\mathbf{n}_{2} = 10$ $\mathbf{n}_{3} = 11$

$\Sigma\mathbf{X}_{1} = 40$ $\Sigma\mathbf{X}_{2} = 41$ $\Sigma\mathbf{X}_{3} = 48$

$\Sigma\mathbf{x}_{1}^{2} = 198$ $\Sigma\mathbf{x}_{2}^{2} = 223$ $\Sigma\mathbf{x}_{3}^{3} = 254$

$\sum \mathbf{X}\mathbf{g} = \mathbf{X}_{1}+\mathbf{X}_{2}+\mathbf{X}_{3}=40+41+48=129$

Calculate the total sum of squares as provided by the formula:

$SStotal = \Sigma x2 - \frac{(\sum \mathbf{X})^{2}}{\mathbf{n}}$

Since $\Sigma\mathrm{x}_{1}^{2}$, 198, $\Sigma\mathrm{x}_{2}^{2} = 223$, $\Sigma\mathrm{x}_{3}^{3} = 254$ and $\Sigma x = 40 + 41 + 48 = 129$

Now substitute in the formula above:

$SStotal = 198 + 223 + 254 - \frac{(192)^{2}}{30} = 675—554.7 = 120.3$

Again, calculate the between-group sum of squares using the formula: $\frac{(\sum \mathrm{X}\mathrm{g})^{2}}{\mathrm{n}\mathrm{g}}$ - \frac{(\sum

\\mathrm{X})^{2}}{\\mathrm{n}}

= \\frac{40^{2}}{9}+\\frac{(41)2}{10} + \\frac{48^{2}}{11}-\\frac{129^{2}}{30}

= \\frac{1600}{9}+\\frac{1681}{10}+\\frac{2304}{11}-\\frac{16641^{2}}{30}

= 177.78 + 168.1 + 209.46—554.7

= 555.l34—554.7 = 0.64

To have, within-group which is sum of squares by the formula:

SSwithin = Σ\\begin{array}{c} \\left[\\mathbf{x}_{\\mathbf{g}}^{2-}\\frac{\\left(\\sum \\mathbf{X}_{\\mathbf{g}}\\right)}{\\mathbf{n}\\mathbf{g}}^{2}\\right] \\end{array}

By substituting in the formula, we have:

SSwithin = Σ\\mathbf{X}_{1}^{2}-\\frac{\\left(\\sum \\mathbf{X}_{1}\\right)}{\\mathbf{n}_{1}}^{2}=198\\hbox{--}\\frac{40^{2}}{9}=20.22+

= Σ\\mathbf{X}_{2}^{2}-\\frac{\\left(\\sum \\mathbf{X}_{2}\\right)}{\\mathbf{n}_{2}}^{2}=223-\\frac{(41)^{2}}{10}=54.9+

= ΣX33-\\frac{\\left(\\sum \\mathbf{x}_{3}\\right)}{\\mathbf{n}3}^{2}=254\\hbox{--}\\frac{\\left(48\\right)2}{11}=44.54

= 20.22 + 54.9 + 44.54 = 119.66

Another approach to obtain within-groups using formula

SSwithin = SStotal—SSbetween substituting we have:

= 120.3—0.64 = 119.66

To obtain the degree of freedom through different sources of variation in the formula as:

Total degree of freedom (df1) = n—1 = 30—1 = 29

Between-groups degree of freedom (dfB) = k—1 = 3—1 = 2

Within-groups degree of freedom (dfw) = n—k = 30—3 = 27

To obtain the variance estimate (mean square), which is between group and within-group. This is carried out by dividing the SSbetween by dfbetween and SSwithin by dfwithin as given by formula:

MSBW =\\frac{\\mathbf{S}\\mathbf{S}_{\\mathbf{B}\\mathbf{W}}}{\\mathbf{D}\\mathbf{F}_{\\mathbf{B}\\mathbf{W}}}=\\frac{0.64}{2}=0.32

MSW = \\frac{\\mathbf{S}\\mathbf{S}_{\\mathbf{W}}}{\\mathbf{D}\\mathbf{F}_{\\mathbf{W}}}=\\frac{119.66}{27}=4.43

F statistics is calculated by using the

F =\\frac{\\mathbf{M}\\mathbf{S}_{\\mathbf{B}\\mathbf{W}}}{\\mathbf{M}\\mathbf{S}_{\\mathbf{B}\\mathbf{W}}}, Now substituting into the formula, we obtain:

F =\\frac{0.32}{4.43}=0.07

Now we shall search for critical value in F-table with degree of freedom for between groups at horizontal across the table, whereas the degree of freedom for within groups at vertical down left side of the table. Given as (between) = 2 and df (within) = 27 at α =.05

Step 4: Decision: since the calculated F value is 0.07 is less than the F (critical) = 3.32 for df = (2, 27). Hence the H0 is retain. That is there is no significant difference among the means of the groups.

Step 5: Conclusion: it is concluded that there is no significant difference among the groups.

This may be by chance or sampling error.

The summary of our analysis can be provided in table as:

Table 11.5: One-way ANOVA Summary Table

Source of Variance

Sum of Squares

Degree of Freedom

Mean Sum of Square

F

Between Groups

0.64

2

0.32

Within Groups

119.66

27

0.07

4.43

Total

120.3

29

Now we have illustrated how to compute t-test, z-test, and ANOVA in this book. However, you will meet other tests such as ANCOVA, MANOVA and MANCOVA in other books.

Students Activity

The following are set of test scores for three sample groups:

S/N

1

2

3

4

5

X1

3

5

4

5

4

X2

3

4

5

6

7

X3

4

4

4

6

8

Determine whether the set of scores are significantly different or not.

Construct an ANOVA for the following data

Method

1

2

3

4

Conventional

7

8

4

9

Discussion

6

6

4

8

Experimental

6

5

4

5

Use the data above to verify a null hypothesis at $\alpha = 0.05$ and state whether the methods have significant differences.

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CHAPTER TWELVE

INFERENTIAL TECHNIQUES II

12.1Introduction

This chapter discuss non-parametric test. Non-parametric tests are tests that do not depend on a knowledge of the population distribution or its parameters. It does not test hypothesis based on the parent population. It requires different assumption and sometimes been referred to distribution free test. It also uses data collected from nominal and ordinal measurement. Non-parametric statistics gives considerable view on the general idea of statistical inference because the perspectives are not clouded in complicated Mathematics. They were developed to deal with situation where the population distribution is non-normal or unknown. However, parametric tests are better than non-parametric test, because they are more likely to reject a false hypothesis or unknown. There are many non-parametric tests, but this chapter will discuss few.

12.2Objectives

At the end of this chapter, you should be able to:

define chi-square test and conditions governing its application.

enumerate the step for computing chi-square test.

carryout chi-square analysis.

define Wilcoxon signed rank test

outline the procedure form using Wilcoxon's signed rank test

demonstrate on computation of Wilcoxon's signed rank test

12.3Chi-Square (x2)

The chi-square is non-parametric test developed by Karl Pearson in 1900. It is used to determine whether or not a significant difference exists between the observed and expected frequencies. Usually, the frequencies are associated with common categorizations such as, Boy or Girls, true or false, agree or disagree, success or no success etc.

The chi-square is the most popular and reliable statistical test that compare observed frequency distributions with theoretical or expected distributions.

12.3.1Basic Conditions for Chi-square

There are four major conditions to be satisfied for using chi-square analysis and they are:

The sample observation must be independent from each other.

The sample data are randomly selected from the population.

Sample size should be fairly large at least between 25 and 250, with not more than 20% of the expected frequency should be less than 5.

Data should be nominal measurement in nature.

The chi-square (x2), the observed and expected frequencies composite in a table known as contingency table. Contingency table has number of rows and column build-up of observed and expected frequencies. The formula for computing chi-square is:

$$x2 = \frac{\left(\mathrm{O}_{\mathrm{f}}-\mathrm{E}_{\mathrm{f}}\right)^{2}}{\mathrm{E}_{\mathrm{f}}}$$

Where Of = is the observed frequency

Ef = is the expected frequency.

12.3.2Computation of Chi-square Test for Goodness-of-fit

Chi-square analysis is conducted for Goodness-of-fit when one sample is measured on one sample. Researcher may be interested to find out when the observed frequencies concerned with or fit some assumed theoretical distribution for the population from which the sample data are selected. The observed frequencies are obtained from empirical observations conducted during research studies, while the expected frequencies are collected on the hypothesis.

Now we shall demonstrate how to calculate chi-square test of goodness-of-fit with the following example 12.1

A descriptive survey was conducted to determine opinion on the political party membership. Use a null hypothesis to determine whether there is statistically significant.

Table 12.1: Data Generated Party Membership

SEX	APC	PDP	SDP	YPP	TOTAL
MALE	20	5	10	15	50
FEMALE	5	20	15	10	50
TOTAL	25	25	25	25	100

Test the political membership with 5% level of significance

The researcher adopts the steps for hypothesis testing in solving this problem.

Step 1: Statement of Hypothesis: Null and alternative hypothesis are:

H0: = there is no statistically significant difference to the views of male and female party memberships.

H1: = There is statistically significant difference to the views of male and female party memberships.

Step 2: Select the level of significance at α =.05

Step 3: Calculate the test statistic: We select the chi-square test of Goodness-of-fit. Since

our sample size is 100 with each category as 20, 5, 5, 20, 10, 15, 15 and 10.

The expected frequency of each category is calculated as:

$Ef = \frac{25\times 50}{100} = 12.5$

$$x2 = \frac{\sum \left(\mathbf{O}_{\mathbf{f}}-\mathbf{E}_{\mathbf{f}}\right)^{2}}{\mathbf{E}_{\mathbf{f}}}$$

Where X2 represent Greek letter for chi-square

Of is the observed frequency

Ef is the expected frequency

Table 12.2: Data for calculation of X2 Goodness-of-fit Statistic.

Of	Ef	Of—E	$\left(\mathbf{O}_{\mathbf{f}}-\mathbf{E}_{\mathbf{f}}\right)^{2}$	$\frac{\left(\mathbf{O}_{\mathbf{f}}-\mathbf{E}_{\mathbf{f}}\right)^{2}}{\mathbf{E}_{\mathbf{f}}}$
20	12.5	7.5	56.25	4.5
5	12.5	–7.5	56.25	4.5
5	12.5	–7.5	56.25	4.5
20	12.5	7.5		

56.25

4.5

10

12.5

–2.5

6.25

0.5

15

12.5

2.5

6.25

0.5

15

12.5

2.5

6.25

0.5

10

12.5

–2.5

6.25

0.5

TOTAL

$\mathbf{x}_{\mathbf{c}\mathbf{a}\mathbf{l}}^{2}$

=

20.0

That is $x^2 = \frac{\sum \left(\mathbf{O}_{\mathbf{f}}-\mathbf{E}_{\mathbf{f}}\right)^{2}}{\mathbf{E}_{\mathbf{f}}}$ = 20.0

Degree of frequency (df) = (4 - 1) = 3

Step 4: Determine the critical region. By doing that you look at table of critical values of

chi-square with df = 3, α = 0.05 and $\mathbf{x}_{\mathbf{v}\mathbf{a}\mathbf{l}\mathbf{u}\mathbf{e}}^{2}$ = 7.82

Step 5: Decision, since the $\mathbf{x}_{\mathbf{c}\mathbf{a}\mathbf{l}}^{2}$ = 20.00 is greater than $\mathbf{x}_{\mathbf{c}\mathbf{r}\mathbf{i}\mathbf{t}\mathbf{i}\mathbf{c}\mathbf{a}\mathbf{l}}^{2}$ 7.82. Hence the null

hypothesis is rejected

Step 6: Conclusion. It is then concluded that there is statistically significant difference to the views of male and female on party memberships.

Another example can be used to illustrate the same procedure for hypothesis testing.

Example 12.2: A survey was carried out to find out the preference mode of some families on the choice of careers for their wards in the table below:

Table 12.3: Data for observed & Expected Frequencies on Careers in Some families

Frequency	Carpentry	Cloth Maker	Engineering	Plumbing	Total
Observed	42	14	24	40	120
Expected	30	30	30	30	120

Analyse the data to determine whether performance mode is significant difference or not.

Steps 1: State the statement of hypotheses

H0: There is no statistically significant difference between the expected and observed preference mode of some families on the choice of careers

H1: There is statistically significant difference between the expected and observed preference mode of some families on the choice of careers

Step 2: Computation and applying the chi-square formula in each cell and add up.

Carpentry = $\frac{\sum\left(\mathrm{O}_{\mathrm{f}}-\mathrm{E}_{\mathrm{f}}\right)^{2}}{\mathrm{E}_{\mathrm{f}}} = \frac{\left(42-30\right)^{2}}{30} = 4.8$ +

Cloth Maker = $\frac{\sum\left(\mathrm{O}_{\mathrm{f}}-\mathrm{E}_{\mathrm{f}}\right)^{2}}{\mathrm{E}_{\mathrm{f}}} = \frac{\left(42-30\right)^{2}}{30} = 8.5$ +

Engineering = $\frac{\sum\left(\mathrm{O}_{\mathrm{f}}-\mathrm{E}_{\mathrm{f}}\right)^{2}}{\mathrm{E}_{\mathrm{f}}} = \frac{\left(24-30\right)^{2}}{30} = 1.2$ +

Plumbing = \\frac{\\sum \\left(\\mathrm{O}_{\\mathrm{f}-}\\mathrm{E}_{\\mathrm{f}}\\right)^{2}}{\\mathrm{E}_{\\mathrm{f}}} = \\frac{\\left(24_{-}30\\right)^{2}}{30} = 3.3

\\therefore Σx2 = 4.8 + 8.5 + 1.2 + 3.3 = 17.8

Step 4: Decision and Conclusion

To decide on the statistical difference of the \\mathrm{x}_{\\mathrm{c}\\mathrm{r}\\mathrm{i}\\mathrm{t}\\mathrm{i}\\mathrm{c}\\mathrn value on single variable. The

degree of freedom given as df = k—1, i.e., 4—1 = 3 with α = 0.05, \\mathrm{x}_{\\mathrm{c}\\mathrm{a}\\mathrm{l}\\mathrm{c}\\mathrm{u}\\mathrm{l}\\mathrı (17.8)

greater than \\mathrm{x}_{\\mathrm{v}\\mathrm{a}\\mathrm{l}\\mathrm{u}\\mathrm{e}}^{2} (7.82), therefore, H0 is rejected.

Sometimes, researcher is to deal with the test of independence were observed and expected frequencies are presented with number of rows and columns (contingency table). Efforts will be directed in that direction in the next unit.

12.3.3Computation of Chi-square Test Independence

To illustrate on how to calculate of chi-square test of independence using the example is shown below:

The Directorate of University Affiliated Programmes, ABU Zaria released their admission list 2021/22 session according to Local Government Area as provided in the Table below:

Compute the chi-square and test for statistical difference at \\alpha = 0.05

DEGREE COURSES

LOCAL GOV'T

MATHS

BIO

PHY

CHEM

ROW TOTALS

Bida

35

40

45

40

160

Edati

30

20

35

35
120
Lapai
20
35
32
30
107
Lavun
25
25
25
20
95
COLUMN TOTALS
110
120
127
125
482

The Researcher has observed the following frequencies through survey research conducted. From the data given chi-square test is mostly appropriate for the statistical test. The formula for chi-test is as: x2 \\frac{\\sum \\left(\\mathrm{O}_{\\mathrm{f}-}\\mathrm{E}_{\\mathrm{f}}\\right)^{2}}{\\mathrm{E}_{\\mathrm{f}}}

Where Of is the observed frequency in each cell of the contingency table Ef is the expected frequency in each cell of the table, then let us determine the expected frequency by using the formula:

Ef = \\frac{\\left(\\mathrm{T}\\mathrm{o}\\mathrm{t}\\mathrm{a}\\mathrm{l}\\mathrm{r}\\mathrm
(\\mathrm{T}\\mathrm{o}\\mathrm{t}\\mathrm{a}\\mathrm{l}\\mathrm{c}\\mathrm{o}\\mathrm
{\\mathrm{G}\\mathrm{r}\\mathrm{a}\\mathrm{n}\\mathrm{d}\\mathrm{t}\\mathrm{o}\\mathr
= \\frac{\\mathrm{F}_{\\mathrm{r}\\times }\\mathrm{F}_{\\mathrm{c}}}{\\mathrm{N}}

Where Ef is the expected frequency
Fr is the frequency of the rth row
Fc is the frequency of the cth column
Applying the formula to calculate the expected frequencies:
1st Row
2nd Row
3rd Row

4th Row

For 1st cell,

$\frac{110\times 160}{482} = 36.52$

$\frac{120\times 160}{482} = 39.83$

$\frac{127\times 160}{482} = 42.16$

$\frac{125\times 160}{482} = 41.49$

For 2nd cell,

$\frac{110\times 120}{482} = 27.39$

$\frac{120\times 120}{482} = 29.88$

$\frac{127\times 120}{482} = 31.62$

$\frac{125\times 120}{482} = 31.12$

For 3rd cell,

$\frac{110\times 107}{482} = 24.42$

$\frac{120\times 107}{482} = 26.64$

$\frac{127\times 107}{482} = 28.19$

$\frac{125\times 107}{482} = 27.75$

For 4th cell

$\frac{110\times 95}{482} = 21.68$

$\frac{120\times 95}{482} = 23.65$

$\frac{127\times 95}{482} = 25.03$

$\frac{125\times 95}{482} = 24.64$

Since the frequencies is calculated, entries are made in contingency Table 12.5

Table 12.5: A 4 x 4 contingency Table Admitted students by Local Gov’t and Academic courses

ACADEMIC COURSES

LOCAL GOV’T

MATHS

BIO

PHY

CHEM

ROW TOTALS

Bida

36.52

39.83

42.16

41.49

160

Edati

27.39

29.88

31.62

31.12

120

Lapai

24.42

26.64

38.19

27.75

107

Lavun

21.68

23.65

25.03

24.64

95

COLUMN TOTALS

110

120

127

125

482

Computation of Degree of freedom

The degree of freedom could be determine using the formula below:

df = (r—1) (c—1)

Where df is the degree of freedom

r is the number of rows

c is the number of columns

Using the above example to determine degree of freedom, for a 4 x 4 contingency table as: df = (r - 1) (c - 1) = (4 - 1) (4 - 1) = 3 x 3 = 9

Adopting the procedure of hypothesis testing using the problem above, we shall now do the chi-square test as:

Step 1: Formulation of Hypothesis

H0: Students' admission into academic course, is not dependent on the local government areas i.e., (x2 = 0)

H1: Students' admission into academic course is dependent on the local government areas i.e., (x2 \\neq 0)

Step 3: Selecting the significance level α =.05

Step 4: Choosing of appropriate statistical test, with the data collected in the form of frequencies and cross-tabulated in a contingency table then chi-square test is

appropriate statistical test for the data in Table 12.4. The expected frequencies are calculated, and chi-square test of independence is calculate applying the formula:

$$x^2 = \frac{\sum\left(\mathbf{O}_{\mathbf{f}}-\mathbf{E}_{\mathbf{f}}\right)^{2}}{\mathbf{E}_{\mathbf{f}}}$$

Where, Of is observed frequencies

Ef is expected frequencies

Table 12.6 shows the calculation of chi-square test of independence.

Table 12.6: Data Calculating x_{test}^{2} of Independence

Cells	(1) Of	(2) Ef	(3) Of—Ef	(4) $\left(\mathbf{O}_{\mathbf{f}}-\mathbf{E}_{\mathbf{f}}\right)^{2}$	(5) $\frac{\left(\mathbf{O}_{\mathbf{f}}-\mathbf{E}_{\mathbf{f}}\right)^{2}}{\mathbf{E}_{\mathbf{f}}}$
1	35	36.52	–1.52	2.31	0.06
2	30	27.37	2.61	6.81	0.25
3	20	24.42	–4.42	19.54	0.80
4	25	21.68	3.32		

11.02
0.51
5
40
39.83
0.17
0.03
0.00
6
20
29.88
–9.88
97.61
3.27
7
35
26.64
8.36
69.89
2.62
8
25
23.65
1.35
1.82
0.89
9
45
42.16
2.84
8.07
0.19
10
35
31.62
3.38
11.42
0.00
11
22
28.19

−6.19

38.32

1.36

12

25

25.03

−0.03

0.00

0.00

13

40

41.49

−1.49

2.22

0.05

14

35

31.12

3.88

15.05

0.48

15

30

27.75

2.25

5.06

0.18

16

20

24.64

−4.64

21.53

0.87

Σx2 = 11.08

\\alpha \\mathrm{f} = (r—1) (c—1) = (4—1) (4—1) = 9

Step 4: Determine the critical value, looking for critical value from the table, while df = 9

at α = 0.5. The critical value is 4.48 and calculated x2 (11.08)

Step 5: Decision. Since the calculated x2 is more than critical value. The null hypothesis is therefore rejected.

Step 6: Conclusion. It is concluded that students' admission into academic course is dependently on the local government area.

Let us summary steps to guide you calculating chi-square Test of independence

Step 1: Write each of the observed frequencies in the appropriate cell

Step 2: Compute the row, column, and grand totals

Step 3: Find Of—Ef and $\left(\mathrm{O}_{\mathrm{f}}-\mathrm{E}_{\mathrm{f}}\right)^{2}$ for each cell.

Step 4: Compute for x2 = $\frac{\mathrm{O}_{\mathrm{f}}-\mathrm{E}_{\mathrm{f}}}{\mathrm{E}_{\mathrm{f}}}$

Step 5: Calculate x2 by adding all the $\frac{\mathrm{O}_{\mathrm{f}}-\mathrm{E}_{\mathrm{f}}}{\mathrm{E}_{\mathrm{f}}}$

Step 6: Find the df using the formula (r—1) (c—1)

Step 7: Look for significance of the computed value of x2 from the table of critical values of x2. If the calculated value of x2 is greater than equals the critical value, the H0 is rejected in favor of H1

12.4Wilcoxon's Matched Pairs of Signed-rank Test

This type of test is another non-parametric test. It operates similar to t–test for dependent (correlated) samples. It is known as Wilcoxon's signed-rank test. It is a test employed for testing null hypothesis and is used to find out the statistical difference between two samples consisting of matched pairs of subjects. Wilcoxon signed test makes use of the difference between pairs of scores.

12.4.1Computation of Wilcoxon's Signed-rank Test

Wilcoxon's signed rank test has the following procedure, performing the test.

Determine the signed difference by matching each pair of observation.

Rank all the differences obtained without respect to sign.

Affix to every rank the sign positive or negative by indicating which ranks rise from differences and which ranks rise from negative differences

Find 'T'; by adding all the ranks which have a positive sign.

Find 'T'; by adding all the ranks which have a negative sign.

Find T, the smallest of T+ and T- forgetting their signs.

Determine the significance of T in any of two approaches, depending on the size of N.

If N is 15 or less that the significance is determine through comparing its values to the tabulated value in the table of critical values in the Wilcoxon signed rank.

If N is more than 15, the significance of Tis determine through the table of normal curve after computing a z-value using the formula:

$$Z=\frac{\mathbf{T}-\frac{\mathbf{n}(\mathbf{n}+1)}{4}}{\sqrt[]{\mathbf{n}\left(\mathbf{n}+1\right)(2\mathbf{n}+1)}}$$

T = the term (out of T+ and T-) with the smallest frequency of occurrence, ignoring their signs

n = the number of non-zero differences.

For N more than 15 could try in next edition or you read from other books.

Example: To test whether the hypothesis from two distributions of scores are identical. Use the matched-pairs data below:

Pairs	X	Y
1	53	69
2	73	60
3	79	64
4	74	66
5	67	46
6	70	65
7	77	59
8	64	68

Solution

Step 1: Find the signed difference between the two scores (see column iv, in Table 12.7).

Step 2: The differences are ranked with respect to sign (see column v in table 12.7)

Step 3: Each rank is affix with the sign (i.e., positive, or negative (see column v in Table 12.7).

Table 12.7: Data Pairs of Scores with Differences and Ranks

i

Pairs

ii

X

iii

Y

Iv

Differences

v

Ranks of difference

Vi

Rank with Less Frequency Sign

1

53

69

\\hbox{--}16

3

3

2

73

60

13

5

3

79

64

15

4

4

74

66

8

6

5

67

46

21

1

6

70

65

5

7

7

77

59

18

2

8

64

68

\\hbox{--}4

8

\\frac{8}{11}

Step 4: Determine T. This is done by adding the rank with less frequent sign. The signed rank is 6, while the negative signed ranks are 2. Hence, the add-up of the two ranks that are negative (8 + 3), since 2 is less than 6. We have T = 8 + 3 = 11.

Step 5: Determine the critical region. Looking at table of critical values in the Wilcoxon signed-rank test when N = 8 less than 15 and T = 11. With \\alpha = 0.05 level of significance is 4.

Step 6: Decision to decide whether to reject or accept the H0. The decision has the following rules:

If T value is less than or equal to the tabulated value, the H0 is retain.

If T value is greater than the tabulated value, the H0 is rejected.

Results obtained are:

Tvalue = 11

Tabulated value = 4 since the values are obtained then H0 is rejected.

Student Activity

The shows the result of the opinion poll concerning the introduction of History combination in College of Education.

Table: Opinions of Lecturers and General Public on History Combination.

Scales

Lecturers

General Public

Row Total

Strongly Agree

25

10

35

Agree

15

10

25

Disagree

05

10

25

Strongly Disagree

05

15

20

Column Totals

50

50

100

If the stated null hypothesis is, there is no significant difference of opinion between Lecturers and General public concerning the introduction of History combination in College of Education curriculum. Does the data support the Hypothesis?

The Provost claimed that his College employed 50% state indigene males, 25% state indigene females, 15% non-indigene males and 10% non-indigene females. To test this claim a Researcher randomly selected 120 employees and obtained the observed frequencies.

Category

Observed Frequencies

Indigene Males

75

Indigene females

30

Non-Indigene Males

18

Non-Indigene Females

12

Test the Provost's claim with a 5% level of significance.

The data provided are set of scores: P1: 12, 11, 10, 9, 7, 6 and 5, P2 = 10, 8, 8, 5, 3, 2. Use the Wilcoxon's signed rank test whether statistical significance.

References

Awotunde, P.O & Ugodulunwa (2002). An Introduction to Statistical Methods in Education. Printed and published in Nigeria by Fab Anieh (Nig). Ltd.

Sambo, A.A (2008). Research Methods in Education Stirling-Horden Publishers (Nig) Ltd