HANDBOOK OF COGNITIVE MATHEMATICS EDUCATION

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THIS IS A PLACEHOLDER. IF YOU WANT TO HAVE AN ACTUAL STATEMENT HERE, YOU HAVE TO MAKE SOME CHOICES USING BOOK'S METADATA MODAL.

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Federal University of Technology, Minna, Nigeria

ISBN:978-978-602-429-5

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TERTIARY EDUCATION TRUST FUND

Book Development Project

The Tertiary Education Trust Fund (TETFUND) has the mandate to establish and nurture the Higher Education Book Development Project in Nigeria. Book scarcity has reached a crisis proportion in the country as evident not only in the quantity of books available but also in the quality of locally produced books. Given the seriousness of the paucity of reading and learning materials in Nigeria's higher educational institutions, the TETFUND Book Development Project is designed to reactivate and nurture research and the publication of academic books and journals in hard and e-forms in Nigerian higher educational institutions, thereby empowering tertiary institutions in Nigeria to benefit from and contribute to knowledge production and nationally and globally. Advancements in science and technology, especially ICT, and the influence of globalization have profoundly transformed the context, from and the scope of knowledge production that Nigerian higher educational institutions should be assisted to fully participate in and contribute to the global system of generating and disseminating knowledge. The uniqueness of the present intervention lies in the fact that through it, TETFund will assist Nigerian higher educational institutions to restore and sustain the capacity for academic publishing.

The promotion of indigenous authorship and the resuscitation of local publishing of books are critical instruments in addressing the dearth of textbooks, including basic test and specialized textbooks in various disciplines in Nigeria's higher educational institutions. Restoring the culture of indigenous authorship and the production of indigenous books would ensure the availability of books that address local needs and reflect familiar realities and experiences.

The book production component is one of the three areas of intervention of the TETFund Book project. The others are the revitalization of academic publishing and the support of academic journals. This first phase of the book production intervention is directed at the production of peer-reviewed basic textbooks written by Nigerian academics for universities, polytechnics and colleges of education and specialized books in various subject areas as well as the publication of books of high-quality PhD theses from Nigerian Universities that have successfully gone through a rigorous assessment process. This would contribute to solving the problem of paucity of books in Nigeria's higher educational institutions.

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6, Zambezi Crescent, Off Aguiyi Ironsi Street,
Maitama, Abuja, Nigeria.

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FOREWORD

I feel greatly honored to have been asked to write a forward for the book 'Handbook of Cognitive Mathematics Education'. Having read the entire content of the book, I wish to assert that 'Handbook of Cognitive Mathematics Education' is truly a guide for Mathematics Education students at NCE and Graduate Levels. The book embodies a good deal of rudimentary knowledge required of a prospective teacher at least but not limited to the foundational (Basic School) level of learning. It entails such vital information as: Teaching and Teacher, Learning and Learner, the enumerated the Nigeria's Philosophy of Education, the objectives of teaching mathematics at the primary and secondary levels, Describe the significant changes to the mathematics curriculum in Nigeria from the pre-independence era to the present, as well as the causes behind these changes. The book also explicitly explained the subtopics relevant to Mathematics Education such as; Mathematics Methodology, Teaching And Learning Of Mathematics, Problems And Prospect Of Mathematics In Nigeria, Innovative Instructional Strategies, Working Tools, Mathematics Laboratory, Geometrical Constructions, Improvisation, Questioning Techniques, Problem Solving, Common Errors In Mathematics, Research Concepts, Research Problem, Research Process, Literature Review, Population Of Study, Instrumentation, Statistical Analysis In Research, Evaluation In Mathematics Education, Classroom Test, Test Development, Classroom Administration And Scoring, Continuous Assessment Educational Goals/Objectives, Approaches, Methodology and Techniques of Teaching, among other, which are essentially knowledge required of a teacher and/or a teacher to be.

The Authors, Dr. Aliyu A. Zakariya (PhD) and Ahmad Manko Umar (PhD) had no doubt exhibited that they are Teachers to the Core and had contributed immensely to the existing body of knowledge in their choosing area of calling. I am confident that in no time we will see more of their efforts.

I have no doubt that the book 'Handbook of Cognitive Mathematics Education' would be of help to Education students generally. I cannot hesitate to recommend this all-important book to all and sundry.

Thank You

Prof. Maruf Olasunkanmi Ibrahim
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ACKNOWLEDGEMENT

We wish to acknowledge those who have contributed to various stages of this book.

Our special appreciation goes to Prof. M.O. Ibrahim, Prof. Mamman Musa, Prof. Y. K. Kajuru, Prof. Inekwe O. and Prof. C. Bolaji and all the authors of the books consulted during the writing of this book.

Our special thanks go to Dr. D. Beji, Dr. Auwal Haruna Gera, Mal. Idris Musa, Prof. Ramatu Gimba and all others not mentioned. May Almighty Allah bless you all for your contributions towards the success of this book.

Thank you all.

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CHAPTER ONE

MATHEMATICS METHODOLOGY

1.1 Objectives

By the end of this chapter, you should be able to:

i. Enumerate the Nigeria's Philosophy of Education

ii. State the objectives of teaching mathematics at the primary and secondary levels

iii. Describe the significant changes to the mathematics curriculum in Nigeria from the pre-independence era to the present, as well as the causes behind these changes.

1.2 Introduction

The philosophy of any endeavor involves its objectives or motivations. As a country, Nigeria aspires to establish a free and democratic society, a just and equitable society, a unified, strong, and self-reliant nation, a great and dynamic economy, and a place where all residents have access to many possibilities. Since the establishment of formal education in Nigeria in 1842, the mathematics curriculum has undergone several changes. The progression begins with the age of formal Arithmetic, Algebra, and Geometry, continues through the era of traditional mathematics and the contemporary mathematics conflict, and culminates in the current day with daily general mathematics. These changes have always been necessary by the necessity for Nigeria's education system to adapt to the new world order of globalization, ICT, and the role mathematics should play in the nation's scientific and technical growth, as well as in response to social needs and expectations.

1.3 Nigeria's Philosophy of Education

Based on these governmental goals, the National Policy on Education (NPE, 2013) outlines the Nigerian educational philosophy as follows:

(a) Education is an instrument for societal change and national progress. Every Nigerian has a right to and is required to receive an education, regardless of gender, social status, religion, ethnic background, or any other unique personal challenges;

(b) education is crucial for the promotion of a progressive and united Nigeria;

(c) education maximizes the creative potentials and skills of the individual for self-fulfillment and general societal development;

(d) education is compulsory; and

(e) education is to be qualitative, comprehensive, functional, and relevant to.

1.4 Objectives of Teaching Mathematics at the Primary and Secondary Levels

General Objectives of Primary Education

i. The development of lifelong reading and math skills as well as the ability to successfully communicate.

ii. The establishment of a solid foundation for analytical and introspective thought

iii. Citizenship education provides a foundation for effective participation in and service to society.

iv. The development of moral character and healthy attitudes

v. Giving the child opportunities to practice manipulative skills that will enable him to function well within the confines of his capacity;

vi. fostering in the child the ability to adapt to his changing environment; and

vii. laying the groundwork for further educational advancement, including preparing him for the local trades and crafts.

From the above broad aims, Abimbade (1995) produced a list of objectives of mathematics instruction in the primary school; they include the following:

i. Provide the kid with the requisite fundamental numeracy abilities

ii. Teach the youngster how to apply these skills to solve issues.

iii. To teach the kid fundamental manipulation abilities necessary in everyday life.

iv. To teach the youngster basic logical reasoning skills.

v. To introduce the youngster to the fundamentals of record keeping and all facets of accounting.

The General Objectives of Secondary Education

Section 4, item 18 of the National Policy on Education outlines the objectives of secondary school education as follows:

a. To increase the number of primary school students who have access to a better education, regardless of their gender, social, religious, or cultural origin.

b. To broaden its curriculum to accommodate the variety of skills, opportunities, and responsibilities possessed by or available to secondary school pupils.

c. To prepare pupils to thrive in our contemporary world of science and technology

d. To cultivate and promote Nigerian culture, art, and language, as well as the cultural legacy of the globe.

e. Raise a generation of individuals who can think independently, respect the perspectives and feelings of others, respect the dignity of work, and recognize the principles outlined in our broad national objectives and live as decent citizens.

f. To promote Nigerian unity by emphasizing the links that unite us notwithstanding our differences.

g. To instill in its students a drive for accomplishment and self-improvement in school and beyond.

h. In accordance with the above method, the Mathematics Association of Nigeria (MAN) determined the following objectives for teaching mathematics at the secondary level at its 1977 conference in Benin:

1. To arouse curiosity in mathematics and to give a strong basis for daily life
2. To acquire computational abilities
3. Encourage the desire and skill to be correct to a degree commensurate with the issues at hand.
4. To cultivate accurate, logical, and abstract thinking
5. To cultivate the capacity to spot issues and solve them using mathematical knowledge.
6. To give the mathematical foundations essential for higher study, and 7) to inspire and foster creativity.

1.5 Mathematics Curriculum Development in Nigeria from Pre-Independence Era – Date.

On September 24, 1842, Thomas Birch Freeman, an English Missionary, landed in Badagery, Lagos, introducing western-style education to Nigeria. Europeans (white males) arrived in Nigeria not only as merchants and adventurers, but also as missionaries. They brought with them a western-oriented education whose primary purpose was to develop Bible readers and catechists (Odili, 2006). The white man needs natives with reading, writing, and arithmetic skills. Consequently, arithmetic was one of the three R subjects introduced in early schools. The mathematics curriculum at this era consisted of simply elementary arithmetic procedures. This just requires the utilization of the four fundamental processes. The selected topics focused mostly on commercial arithmetic involving percentages, ratios, proportions, profit and loss, interest, discount, and basic measurement. In addition, the curriculum stressed the memorization of multiplication tables and formulas. The publications utilized were entirely of foreign origin: efficient arithmetic, A- Shelling Arithmetic, and Larcombe Arithmetic (Fajemidagba, 2001).

In the southern Christian Missionary Schools, the notion of mathematics as a school topic had formed by the early 1950s, but the subject was taught in three distinct sections:

arithmetic, algebra, and geometry. The books usually utilized were Durell, and Channon and Smith, each in 3 – volumes. In 1956, the West African School Certificate Examination includes mathematics as a single subject (WAEC). But arithmetic survives as a standalone topic from mathematics in teacher training institutes.

As a consequence of the Soviet Union launching its first earth-orbiting satellite (Sputnik) in space in November 1957, there was a desire for re-evaluation and reform of the global shift in school mathematics curriculum and program in Europe and America. It was believed that mathematics was the foundation of this technical achievement. Significant shifts from the meaningful arithmetic period to a modern mathematics curriculum with a focus on technology were observed during this time. In 1958, the American School Mathematics Study Group (SMSG) Project, in 1958, the University of Illinois Committee on School Mathematics (UICSM) Project, in 1959, the Greater Cleveland Mathematics Programme, and in 1962, the British School Mathematics Project were all established as a result of the reform. 2012 (Awofala).

At the International Conference on Science in the Advancement of New States at Rehovoth, Isreal, in July, 1960 a plea was made for curriculum reform throughout all of Africa. In 1961, at the Endicott House Conference held in Delham, Massachusetts, the African Education Programme was started in response to the Rehovoth request (AEP). Nigeria participated in conference. The purpose of AEP was to introduce current technique of curriculum creation and adaptation to African's demands in the fields of languages, mathematics, science and social studies. The African mathematics programme (AMP) was launched in 1962 with workshops held in Entebbe, Uganda and Mombasa, Kenya. AMP aimed to improve the quality of mathematics teaching and to develop in each participating country a cadre of knowledgeable and competent modern mathematics teachers who could lead modernization movements in their respective countries. Other aspects of the initiative focused on the creation of appropriate publications, audiovisual aids, and testing materials. African Mathematics Programme (AMP) was tasked with the preparation and production of teaching textbooks for use in schools, teacher training institutions, and in-service institutions. It was also tasked with the trial teaching and testing of the already produced materials in various schools and colleges, as well as the quantitative and qualitative training of teachers and tutors in the proper use of the materials produced. 54 participants from 13 countries, including Nigeria, attended the first writer's workshop in Entebbe. Between 1962 and 1969, it hosted intense residential workshops and generated more than 80 mathematical texts for elementary and secondary schools, as well as teacher training and sixth form colleges.

The introduction of AMP materials into Nigeria in January 1964 marked the beginning of the modernisation of mathematics. Professor Grace Alele Williams directed the most successful AMP project in Nigeria, which was the Lagos experiment. Grace Alele Williams said that the Lagos pilot study on contemporary mathematics done between 1964 and 1968 was a tremendous success, but this could not be affirmed for other states in the federation. Some Ghanaian school instructors introduced the joint

school project (JSP) textbooks as an additional component of the modernisation of the mathematics curriculum.

One of the major events in the history of education in Nigeria was the momentous national conference of curriculum in September, 1969. The meeting was called to re-orient the structure and curriculum provision of Nigeria education to satisfy:

i. The demands of adolescents and adults in Nigeria;

ii. The social-economic needs, values, aspirations, and growth of the Nigerian society; and

iii. The curriculum substance, the topic material, which is the means to achieve the goals. In 1969, the Nigeria Educational Research Council (NERC), which is now the Nigeria Educational Research and Development Council (NERDC), was tasked with creating a new mathematics curriculum (Modern Mathematics).

By January 1971, contemporary mathematics had been implemented in all Lagos elementary schools. The secondary mathematics curriculum was also updated to emphasize the relevance of mathematics. People began to discuss the 6 – Ms of modern mathematics, 'Modern Mathematics enhances the significance of Mathematics. Among the objectives of contemporary mathematics are:

§ Encouraging youngsters to love and comprehend mathematical concepts and principles

§ To guarantee that there is no significant difference in the language of mathematics in elementary and secondary schools, or between secondary and post-secondary schools.

§ To familiarize children with the mathematics encountered in everyday life and to stress their applicability to current practical issues; and - To strengthen the students' problem-solving skills (Odili, 2006).

§ By 1974, contemporary mathematics was taught in nearly all federated nations. The activities of Curriculum Development Agencies supplemented those of NERDC. In 1974, the West African Examination Council (WAEC) incorporates current mathematics into the Schools Certificate Examination. However, the outcome was subpar. In 1975, individuals began to weep at the widespread failure in mathematics. Numerous urged for the eradication of modern mathematics, which was the subject of many accusations.

§ On the 6th and 7th of January, 1977, the Federal Minister of Education invited famous mathematics educators, mathematicians, and others who contributed significantly to the development of mathematics education in Nigeria to a conference in Benin. According to the letter issued to the attendees, the primary goal of the conference was for the participants to advise the government on the content and technique of school mathematics. In his lecture, the then-federal commissioner of education, Dr. (Col.) A.A. Ali, proclaimed the elimination of the teaching of current mathematics without consulting specialists.

§ In response to the eradication of contemporary mathematics, the National Task Force established by the NERC in July 1976 to explore the difficulties facing the teaching

of mathematics in Nigerian schools and colleges convened at the University of Ibadan in February 1977. The Task Force's terms of reference were as follows:

1) To examine the different aspects of the controversy over the teaching of modern' or 'traditional' mathematics in Nigerian schools;

2) To examine the existing school mathematics curricula in light of this controversy and highlight the problems and issues; and

3) To make concrete proposals for the development of appropriate mathematics curricula for the various levels, including implementation suggestions.

§ The plenary session was followed by the formation of two working groups, one on primary and teacher education and the other on junior and senior secondary. These two groups worked independently on curriculum, instructor, teaching aids, and school administration for their respective grade levels. The adoption of the Task Force's recommendations by the government resulted in revised curricular standards for the relevant grade levels (Awofala, 2012).

§ National Council on Education (NEC) had a meeting in October 1977 to coordinate the operations of the NERC and Comparative Education Study and Adaptation Centre (CESAC). In March of 1978, these diverse groups presented their work at a national criticism workshop held in Onitsha. The conference adopted the NERC primary syllabus and curriculum with only minor modifications, while the CESAC secondary syllabus was reworked to conform to the new structure of the 6- 3-3-4 system as outlined in the National Policy on Education (FRN, 1977) and divided into two syllabi: the junior secondary mathematics syllabus and the senior secondary mathematics syllabus. 1978 was the year that the Federal Ministry of Education approved the revised curricula. The curriculum was a balance between "traditional" and "contemporary" mathematics. Whereas "modern" mathematics focused the production of concepts and logical reasoning, "traditional" mathematics valued manipulative abilities. The formal implementation of the primary mathematics curriculum began in 1979, while the junior secondary mathematics curriculum was officially presented to junior secondary schools in 1982 and has been in use without significant revisions throughout the years.

The Federal Government of Nigeria's 1999 implementation of the Universal Basic Education (UBE) Programme as an educational reform program paved the way for a comprehensive revision of the elementary and secondary school curriculum. The re-alignment of junior high school with elementary school is one of the fundamental characteristics of the UBE plan. This disarticulation initiative, along with other main objectives, demanded a revision of the current primary and junior secondary school curriculum to accommodate the 9-year basic education program. Changes to the elementary and junior secondary mathematics curricula are a component of this broader reform initiative. The UBE program represents the Nigerian government's plan for implementing the Education for All (EFA) deal. The objectives of UBE as stated in the UBE Act of 2004 are to: (1) Develop in the entire citizenry a strong consciousness for

education and a strong commitment to its vigorous promotion; (2) Provide free, universal basic education for every Nigerian child of school-going age; (3) Dramatically reduce the incidence of drop-out from the formal school system (through improved relevance, quality, and efficiency); and (4) Cater for the learning needs of young persons, who for one reason or another are unable to attend formal school. The curriculum revision began in 2001 and lasted until 2007, when the Countrywide Council of Education authorized a revised nine-year basic education curriculum for national implementation in 2008. Awofala (2012) identified many important reasons for the modification of the elementary and junior secondary curriculums. These include:

a. satisfying the Universal Basic Education Programme's demands

b. advocating for National Economic Empowerment and Development Strategy (NEEDS), Education For All (EFA), and Millennium Development Goals (MDGs)

c. the requirement to develop better-informed, Information and Communications Technology (ICT)-compliant, bilingual, high-ethical-standard citizens;

d. faults with prior curriculum; and

e. keeping up with emerging global and national concerns.

Among the benefits of the new curriculum are the following:

§ It enables all teachers to teach the same topic in detail;

§ It defines the objectives in learner- performance term and reduces both content and objective to classroom learning experiences for learner;

§ It enables the teachers them to order instruction and work toward the realization of clearly stated performance objective;

§ It includes detail of teacher's activities and pupil's activities guide;

§ The learning activities contained in it include inquiry, research and practical work experiment (NERDC, 2004).

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CHAPTER TWO

TEACHING AND LEARNING OF MATHEMATICS

2.1 Objectives

By the end of this chapter, you should be able to:

i. Discuss the educational applications of the works of some mathematics psychologists such as: Piaget, Bruner, Gagne and Dienes.

ii. Discuss methods of teaching of concepts, principle, skill and proofs

iii. deductive, analytic and synthetic approaches in mathematics teaching

2.2 Mathematics Psychologists

2.2.1 Jean Piaget's Theory of Cognitive Development

According to the idea of cognitive development proposed by Jean Piaget, the intellect of children increases with age. Children progress through a series of cognitive stages that are impacted by both intrinsic abilities and environmental influences. According to Jean Piaget's theory of cognitive development, children go through four stages of intellectual development that correlate to the increasing complexity of their thinking with age. Each kid goes through the stages in the same order, despite the fact that biological maturation and interactions with the environment determine how children develop. Each stage of development entails a separate intellect, with each child's thinking at each level being qualitatively distinct from the others.

Stages of Cognitive Development

Although no phase may be skipped, individual differences in the rate of a child's growth imply that some children may never reach the later phases. Despite the fact that descriptions of the phases occasionally provide an indication of the age at which the average kid would reach each stage, Piaget did not claim that a certain stage was reached at a specific age.

The Sensory motor Stage (Birth to 2 Years)

Children gain knowledge of the world through their perceptions and actions. During this period, a number of cognitive talents arise. These include self-recognition, delayed imitation, object persistence, and representational play. By the end of this stage, children will have proved via play that they can substitute one object for another. Because people

discovered that words could convey both things and feelings, language began to develop. The youngster can now store, retrieve, and categorize information about the outside world.

The Concrete Operational Stage (7 - 11 Years)

At this age, children begin to think rationally about genuine events. Children begin to comprehend the concept of conservation, which asserts that while the appearance of something may change, certain properties stay constant. Children are also capable of psychological turnarounds (e.g. picture a ball of plasticine returning to its original shape). During this stage, children grow less egocentric and begin to consider the feelings and thoughts of others.

The Formal Operational Stage (12 and above)

Physical and perceptual restrictions have no bearing on formal operational reasoning. During this period, teenagers may comprehend abstract concepts (e.g. no longer needing to think about slicing up cakes or sharing sweets to understand division and fractions). They may follow the structure of an argument without considering particular instances. Teenagers are capable of addressing hypothetical situations with several potential answers. For example, if asked, "What would happen if money disappeared in one hour? They may guess on several potential outcomes. Children of around 12 years old can follow the structure of a logical argument without regard to its substance. People gain the ability to think about abstract concepts and test hypotheses rationally throughout this stage. This stage saw the birth of scientific thought, with problem-solvers proposing abstract theories and hypotheses.

2.2.2 Jerome Seymour Bruner

Jerome Seymour Bruner (1 October 1915 – 5 June 2016) was a renowned American developmental and cognitive psychologist. He blends psychological research and classroom practice in his work. Bruner contends, according to Febrianti and Purwaningrum (2021), that mathematics is a science that can be studied through the ideas and structures that already exist in mathematics. Using these mathematical concepts and structures, a search may be conducted for the material's relationships. Bruner claimed that the most crucial aspect of learning is how individuals actively select, retain, and modify knowledge.

Bruner believed that there are three parallel processes involved in learning. The three stages are as follows: (1) acquiring new information, (2) altering information, and (3) assessing the relevance and validity of knowledge. Bruner identifies three stages of child development: enactive, iconic, and symbolic. In the enactive period, toddlers learn through physically interacting with objects. During the iconic stage, children's activities

evolve and lead to more abstract concepts. At this level, there is a conceptual process of imagining an item, but no actual manipulation. In the third stage of symbolic development, the kid manipulates the symbol without reference to things. Bruner develops four learning theories: construction, notation, contrast and variation, and connectedness (Tampubolon, 2018).

Construction:

According to this notion, the best approach for a learner to begin learning mathematical concepts and principles is to create them. To create an idea or principle is to reduce the complexity of the concept or principle by analyzing its constituent pieces. Bruner contends that "any idea, topic, or body of information may be given in a way that is simple enough for any student to comprehend" (Bruner, 1996). According to Bruner, the concept of a concept is based on actions using concrete objects in the early phases of a student's learning. The consequences of this approach for mathematics education are that the logical presentation of new concepts is improper. This is plausible if considering Bruner's proposed indicator phases, enactive, iconic, and symbolic.

Notation:

According to the Notation theory, the initial construction is cognitively simplified and better understood by pupils if it is constructed according to a notation that corresponds to the degree of intellectual development of the learner. The impact of this theory on the teaching of mathematics is the use of developmentally appropriate notation for both ideas and principles. A idea's notation should at least indicate one concept and not another. The use of notation that is inconsistent with the degree of intellectual development of the learners will disturb their comprehension.

Contrast and variation

The theory of contradiction and variation implies that the process of learning from tangible to abstract mathematical concepts must be incorporated into the contradictions and variations (Alamian & Kazemi, 2020). Consequently, as students learn mathematical concepts, the examples must change so that their comprehension is enhanced. The application of this principle to mathematics education is that while introducing a concept, its opposite must be introduced. Moreover, examples and non-examples used to illustrate a concept or idea must differ.

Connectivity

According to the connectedness hypothesis, every mathematical notion, structure, and talent is related to other concepts, structures, and skills. Although the explanation of the

idea or principle must be related to the preceding concept or principle, it need not be linked to notions that are too distant. The application of this idea to the teaching of mathematics is to illustrate the previous notion or principle prior to explaining the new concept or principle. In addition, the explanation or evidence of the notion or principle is provided. The notion or principle may be illustrated by both examples and non-examples.

Bruner established three cognitive representation phases.

The first level is enactive, which is the expression of knowledge through actions. This phase includes the encoding and storing of data. There is no internal representation of the objects involved in the direct handling of the objects. Learning should begin with direct manipulation of items. For example, in mathematics class, Bruner supported the use of algebra tiles, money, and other materials that might be controlled.

Iconic: Which is the visual summary of pictures that occur between the ages of one and six. This level comprises an internal visual representation of external objects as a mental picture or icon. After a student has had the chance to actually manipulate the items, they should be encouraged to create visual representations, such as a form or a diagram.

Here, words and other symbols are used to depict experiences symbolically. Beginning around the age of seven, information is stored in the form of a code or symbol, such as language. Each symbol has a definite relationship to the item it symbolizes, and a learner eventually comprehends the relationship between symbols and the things they stand for. For example, a student in mathematics learns that the plus symbol (+) indicates to add two numbers together while the negative sign (-) means to subtract.

Bruner asserts that children actively engage in learning at a level commensurate with their cognitive development. To enhance the learning experience, educators should optimize the style of presentation rather than the material being taught.

Applications of Brunner's Theory to Education

Bruner felt that by adjusting educational techniques to the cognitive functioning level of children, every topic that can be taught to adolescents and adults can be taught to children. Any topic may be taught to any child at any developmental level.

1. Bruner considered the kid as a minor authority. According to him, a youngster may comprehend the action of disciplines at nearly any age. According to him, the primary difference between a child's and an adult's cognitive processes is quantity, not kind or quality.

2. Every domain of knowledge may be represented as one of three motor, visual and symbolic systems. Teaching every specific topic to any specific individual

necessitates one of the three approaches. Bruner stated that, wherever feasible, a teacher should deliver his or her instruction in all three practical, visual, and symbolic systems.

3. Bruner defends the guided discovery technique as opposed to the independent discovery method. Guided discovery is a manner in which learners are encouraged to grasp and realize the questions with the help of the teacher.

4. Encouraging pupils to grasp the subject's essence: According to Bruner, discovery is an interior concept and process. Bruner argued that this internal process requires a rearrangement of the thinking system. To do this, students should be encouraged to acquire a subject's or field of knowledge's principles.

5. The learning atmosphere should be devoid of worry and tension. The educational environment should be structured so that students may freely express their thoughts, consider a variety of subjects, and arrange their mental conceptions in order to enhance their cognitive ability.

Bruner had the following opinions regarding education and learning:

§ He felt that education should encourage the growth of problem-solving abilities via inquiry and discovery.

§ He felt that subject matter should be given in terms of the child's worldview; • Curriculum should be structured so that the mastery of one skill leads to the acquisition of a more advanced skill.

§ He also promoted teaching through the organization of concepts and learning through exploration.

§ Lastly, he felt that culture should affect the ideas that individuals use to arrange their perspectives of themselves, others, and the world in which they live.

2.2.3 Zoltan Dienes

Zoltan Dienes (1916-2014) was a world-renowned Hungarian mathematician and education psychologist who felt that mathematical structures could be effectively taught to children in the primary grades via the use of manipulatives, games, and tales. Dienes discovered that creating game rules that correspond to the rules found in mathematical systems capitalizes on children's innate propensity for game-based learning. Dienes also discovered that manipulatives were an efficient technique to explain complicated mathematical topics. He created Base 10 blocks (commonly referred to as Dienes blocks) to assist children understand the mathematical foundations in an appealing manner. Base ten blocks, also known as multi base arithmetic blocks (MAB) or Dienes, are mathematical manipulatives (wooden or plastic cubes, rods, and flats) used to teach pupils basic mathematical concepts such as addition, subtraction, number sense, place value, counting, and number bases.

Here are three reasons why Dienes regarded games to be an excellent educational tool.

1. Games enhance the enjoyment and motivation of learners.

Dienes emphasizes in his work that children do not need to reach a particular developmental stage in order to feel the joy of mathematics. The most important thing is that youngsters learn how to think.

Teaching mathematics with games helps engage students who find the subject intimidating and dull. Understanding mathematical patterns and relationships can be an exciting and inspiring experience for youngsters when they learn via games.

2. Games increase learners' problem-solving skills

Fun board games like Snakes and Ladders, Cards, Monopoly, and Scrabble hide mathematical structures and rules that even young children can use to develop higher-level problem-solving abilities like trial-and-error techniques, task simplification, pattern-finding, hypothesis formation and testing, reasoning, and proving and disproving. Games may also be a helpful evaluation tool. It is feasible to more accurately gauge the learners' present level of understanding by paying attention to the strategies they choose throughout the game.

3. Games facilitate the practice and reinforcement of mathematical abilities

Games not only aid in the development of learners' problem-solving skills, but also provide them opportunities to practice and reinforce their mathematical abilities. Learners flourish when they have a specific objective to achieve, even if the route to success is difficult. Children are allowed to consider all possible tactics, and if one fails, they are encouraged to try again. When children receive positive comments for their hard work, they obtain a sense of hope and the realization that they can learn and develop as they face new problems. Mathematical games combined with the use of manipulatives promote and deepen students' thinking without requiring tedious memorization.

2.3 Dienes' Six Learning Phases

Dienes reconstructed Piaget's well-known four-stage process of conceptual development into a six-stage process pertaining to the production of mathematical concepts.

1. *Free Play:* a requirement or beginning point for higher abstractions, involving the acquisition of concrete experiences about the world, its objects, and their relationships through "trial and error"

2. *Rule-based Games:* methodical finding of regularities, rule-invention, learning to play by the rules, making differences between the beginning state and the final state, the rules to be satisfied, and the conditions to be met.

3. *Comparative Structuring:* a discussion of the games, a comparison of the rules, and a search for commonalities among rule-based game structures, excluding specific

elements. Searching for the "common core" of comparable games as their structural (mathematical) content and presenting dictionaries for shared characteristics.

4. *Representation:* diagrammatic, visual, or multimodal portrayal of the abstracted characteristics of the games, extracting the essence of the communalities and mapping the rules and regularities of the actual games to the representations.

5. *Symbolization:* analysis of the representation, study of the cleanable (not comparable) properties of the games as classes of regularities, verbal description of the extracted rules by introducing symbols for the "map" that represent abstract components, and verification of the outcomes of abstract rules in concrete games.

6. *Formalization:* discovery of relationships between the described and symbolically represented qualities of the representation, attaching methods for deducing further properties, finding of descriptions that imply other descriptions, and determination of the rules of deduction. Beginning the process of picking axioms, locating theorems, and developing proofs.

2.3.1 Inductive Method

The Inductive Method is founded on the induction principle. Induction is the process of establishing a universal truth by demonstrating that if it is true for a given case and a sufficient number of other situations, then it is true for all such circumstances. In this technique, an issue is initially answered based on the learner's prior knowledge, thinking, reasoning, and intuition. At this point, he is unaware of any formula, theory, or approach for addressing the problem. When learners are given with a sufficient number of comparable instances, facts, or objects, they attempt to draw a conclusion for each. Consequently, they arrive at a generalization or deduce a formula by a persuasive process of reasoning and solving several situations with comparable characteristics.

Consequently, it is a way for developing a formula using a sufficient number of concrete instances. Thus, the inductive technique of instruction guides us from the known to the unknown, the specific to the general or the example to the general rule, and from the concrete to the abstract. When a number of concrete situations have been comprehended, the student can try generalization.

Example 1: Square of an odd number is odd and square of an even number is even.

Solution:

Particular concept:

$1^2 = 1$ $3^2 = 9$ $5^2 = 25$…..i

$2^2 = 4$ $4^2 = 16$ $6^2 = 36$ …..ii

General concept:

From i and ii, we get

Square of an odd number is odd and Square of an even number is even.

Example 2:

Sum of two odd numbers is even

Solution:

Particular concept:

1+1=2

1+3=4

1+5=6

3+5=8

General concept:

In the above we conclude that sum of two odd numbers is even

Example 3 :

Law of indices $a^m \times a^n = a^{m+n}$

Solution:

We have to start with $a^2 \times a^3$ = (a x a) x (a x a x a)

= a^5

= a^{2+3}

$a^3 \times a^4$ = (a x a x a) x (a x a x a x a)

= a^7

= a^{3+4}

Therefore, $a^m \times a^n$ = (axax….m times)x(axa …n times)

$a^m \times a^n$ = a^{m+n}

MERITS

§ It enhances self-confident.

§ It is a psychological method.

§ It is a meaningful learning

§ It is a scientific method.

§ It develops scientific attitude.

§ It develops the habit of intelligent hard work.

§ It helps in understanding because the student knows how a particular formula has been framed.

§ Since it is a logical method so it suits teaching of mathematics.

§ It is a natural method of making discoveries, majority of discoveries have been made inductively.

§ It does not burden the mind. Formula becomes easy to remember.

§ This method is found to be suitable in the beginning stages. All teaching in mathematics is conductive in the beginning.

Demerits

§ Because some complex and difficult formulas can't be constructed, the applicability of this approach to all topics is constrained.

§ It is a tedious and time-consuming process.

§ Its length is one.

§ Its scope of applicability is somewhat narrow.

§ Because the generalization drawn from a few focused examples may not hold true in all circumstances, inductive reasoning is not completely conclusive.

2.3.2 Deductive Method

In this method, we move from the general to the specific and from the abstract to the concrete. At first the guidelines are taught and then students are requested to use these principles to answer further problems. This method is mostly utilized in Algebra, Geometry, and Trigonometry since these subdisciplines of mathematics employ distinct relations, principles, and formulas. In this method, mathematical assumptions, postulates, and axioms are utilized.

Example 1:

Find $a^2 \times a^{10} = ?$

Solution:

General: $a^m \times a^n = a^{m+n}$

Particular: $a^2 \times a^{10} = a^{2+10} = a^{12}$

Example 2:

Find $(102)^2 = ?$

Solution:

General: $(a+b)^2 = a^2+b^2+2ab$

Particular: $(100+2)^2 = 100^2 + 2^2 + (2 \times 100 \times 2)$

$= 10000+4+400= 10404$

Merits

§ It is short and time saving method.

§ It is suitable for all topics.

§ This method is useful for revision and drill work.

§ There is use of learner's memory.

§ It is very simple method.

§ It helps all types of learners.

§ It provides sufficient practice in the application of various mathematical formulae and rules.

§ The speed and efficiency increase by the use of this method.

§ Probability in induction gets converted into certainty by this method.

Demerits

§ It is not easy to understand.

§ It taxes the pupil's mind.

§ It does not impart any training is scientific method.

§ It is not suitable for beginners.

§ It encourages cramming.

§ It puts more emphasis on memory.

§ Students are only passive listeners.

§ It is not found quite suitable for the development of thinking, reasoning, and discovery.

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CHAPTER THREE

PROBLEMS AND PROSPECT OF MATHEMATICS IN NIGERIA

3.1 Objectives

By the end of this chapter, you should be able to:

i. Identify some problems confronting the teaching of mathematics.

ii. State some factors that are responsible for students' poor performance in Mathematics

iii. Mention some ways of resolving these problems

3.2 Introduction

Mathematics is a subject with numerous aspects. As a language, a certain type of logical structure, a body of information about number and space, a study of patterns, a set of procedures for drawing conclusions, and the essence of our understanding of the physical world, mathematics is seen in several ways. Educators of mathematics asserted, among other things, that no other subject offers a wider range of applications than mathematics (Kankia, 2008). It is the hub around which all other subjects revolve. It is the most important tool for comprehending and investigating our scientific, technical, economic, social, and informational universe. However, it is pitiable and a thing to be lamented about, that this all – important Queen, King and Servant is least desired by the learners. The performance of learners in this all-embracing subject is very poor at all levels. Several reasons have been proffered as the causes of this problem. Pool of studies revealed among others; inadequate qualified teachers, poor quality of instructional technique employed by teachers, and poor students' attitude to mathematics (Eze, 2008; Bawa & Abubakar 2008).

3.3 Problems of Teaching and Learning of Mathematics

According to Odili (2006), the problem of teaching and learning of mathematics can be broadly categorized into three (3) main issues:

- Mathematical Issues
- Pedagogical Issues
- Psychological Issues

3.3.1 Mathematical Issues

The mathematical issues include content, curriculum integration and understanding issues.

3.3.1.1 Content Issue

According to Adepoju in Odili (2006), the mathematical curriculum is technically perfect in terms of its content and objectives. However, two silent extremes appear in the classroom while teaching Mathematics, the topics is either not finished or rushed. At one extreme are teachers that believe that since the curriculum have been broken into yearly, termly and learning unit plans, topics meant for each week or term must be finished before the term runs out. This leads to rushing their lessons at the expense of understanding. At the other extreme, are teachers who feel that rushing will deter students' or pupils' understanding, they therefore, ending up without completing the term or year work. The implication is that the learners may lack the entry behavior required in another lesson or class. To resolve this problem, it is necessary for the classroom teachers to be involved at every stage of curriculum development. Since those teachers have direct contact with learners, their experience in dealing with them over the years will give them the upper hand to suggest the content, scope and time for each subject matter to be included in the curriculum.

3.3.1.2 Curriculum Integration

The problem of mathematics curriculum integration is two-dimensional; the vertical and horizontal (Odili, 2006). The problem of vertical Integration arises when a year's work was not completed before proceeding to the subsequent year. This will lead to creation of gaps within class level. The current primary and secondary mathematics curricular are sequential and spiral in nature. This means that students 'understanding of Senior Secondary mathematics is linked and dependent on his/her understanding of both primary and Junior Secondary Mathematics (Kankia, 2008). Another problem is when teachers fail to relate mathematics to other subjects such as Chemistry, Physics, Biology etc. and everyday use of mathematics in work places, business and industry. They are creating a horizontal curriculum integration problem. To resolve this problem, teachers should make sure that they cover the stipulated content in each class beginning from Primary School.

However, it revealed that Nigerian Primary School is more of a sole teacher, that is a generalist 'who teaches all subjects in the curriculum'. Zakariyya (2009), suggested, that Primary mathematics content should be included in our National Certificate Education (NCE) programme. It should be noted that any single topic not taught in one class poses a learning problem in the next class.

3.3.1.3 Understanding Issue

According to the National Council of Teachers of Mathematics (NCTM, 2000), current learning perspectives include the following assumptions:

- Learning is a process of knowledge construction, not knowledge recording or observation;
- Learning is knowledge-dependent; people use existing knowledge to construct new knowledge; and
- According to (Odili, 2006), most students in JSS III can solve the equations:

  $2x + 3y = 45$

  $2x + 4y = 74$

But the same set of students could not solve the same problem when stated in word. This was as a result of students' inability to relate or see inter-connection of the various processes they know or have learnt in the class. Therefore, teachers should try to have a classroom where mathematical reasoning is encouraged away from a classroom where memorization of procedures is the order of the day. Similarly, they should have a classroom in which conjecturing, inventing and problem solving are integrated in the lesson away from a classroom which emphasizes on mechanistic answer finding.

3.3.2 Pedagogical Issues

The pedagogical issue will be discussed under the following heading; instructional method/procedure, instructional material, classroom organization and teachers.

3.3.2.1Instructional Method/Procedure

Numerous teachers have attempted and continue to teach mathematics using the typical drill and practice method, which emphasizes memorizing of facts and procedural practice on calculation. This standard method should be replaced with methods that enable youngsters to think critically and find things out on their own. The constructivist method to teaching and learning is one of the most prominent approaches to mathematics and science education (Zakariyya, 2008). In constructivist theory, learning is contingent on how each individual learner interprets a given circumstance and derives their own conclusion.

3.3.2.2 Instructional Material

Several research have been conducted to discover the causes of low mathematics ability. Some studies found, among other things, an insufficient supply of certified mathematics teachers, a shortage of instructional tools, and a negative attitude among pupils (Bawa & Bala, 2008). Teaching aids or instructional aides are tangible tools or products designed

to enhance the learning process. Through the active participation of the learner, they make the teaching and learning of mathematical concepts and abilities more understandable. Each learner need independent manipulation materials. The demonstrations of the instructor or a single student are insufficient. With the use of manipulative tools, students' interest in mathematics will be stimulated. The Government, voluntary organizations, and the Parent Teacher's Association should develop mathematics laboratories in elementary and secondary schools. Teachers should be encouraged to create their own educational resources for mathematics class.

3.3.2.3 Classroom Organization

According to NPE (2004), for successful teaching and learning, the teacher-to-student ratio should be 1:35. However, it is a well-known problem that the majority of our elementary and secondary schools, especially those in metropolitan areas, are overcrowded, with an average of 60 to 80 pupils per class.

- In a small class, students will receive more individualized attention, written assignments, creative activities, and field trips, while the teacher will have more opportunity to monitor each student's progress, assign more problem-solving activities, projects, and essay exams.
- Smaller classes produce the necessary, but not sufficient, conditions for successful teaching and learning.

However, big class sizes have the consequence of reducing instructors' morale and increasing their stress levels.

3.3.2.4 Teachers

It has been said that "no educational system can surpass the caliber of its instructors." A teacher is an individual who has had recognized professional training in education at an adequate level and is capable of transmitting information, attitude, and skills to the learner. The NPE (2004) provides for the employment of mathematics specialists with a minimum level of National Certificate of Education (NCE). Recent studies (Idehen, 2006, and Kankia, 2008) indicate that there are insufficient instructors for all disciplines in elementary schools, and the majority of the existing teachers are not particularly qualified. The results also indicate that the majority of primary school teachers in Nigeria are generalists who teach all curricular areas rather than specialists (who has an in-depth knowledge in a subject and is professionally trained and competent to teach the subject). In this way, mathematics is the subject most affected in both Primary and Secondary schools. It is claimed that a person cannot provide what he or she lacks.

The following are potential remedies:

§ NPE (2004), provision for utilization of specialist teachers of mathematics should be implemented;

§ Team teaching should be encouraged in areas where some individual teachers may find it difficult to teach;

§ Mathematics programmes in teacher training institutions should include the content of the Primary School Syllabus;

§ Teacher training institutions should include some new heuristics for teaching mathematics, such as concept mapping and metacognitive strategies.

3.4 Psychological Issues

The following headings will be used to address psychological issues: individual differences and mathematical mindset.

3.4.1 Individual Differences

Different individuals primarily absorb and process information in a variety of ways. Through seeing and hearing, reflecting and acting, logical and intuitive reasoning, analysis and visualization. Cognitive or learning style refers to these distinct methods of acquiring and processing information (Zakariyya, 2009). According to him, learning style consists of three key elements: information processing, instructional preference, and learning method. Information processing is a person's intellectual approach to the processing of information, instructional preference is their preferred learning environment, and learning techniques are the tactics they employ when studying. Individual variations in learning potential and performance can be attributed to mental aptitude, the capacity to reason or think reflectively and to solve issues, according to Odili (2006). This ultimately results in the categorization of pupils or students. Individual differences are an essential component to consider when designing education, given that individuals see and process information differently.

Here are some strategies for accommodating individual differences:

Individually or in small groups, students should do a variety of assignments.

§ Ideally, instructors or teachers should examine the learning styles of their students and design lessons appropriately.

§ Instructors should change their educational strategies in order to assist a variety of students

§ Sorting students into classes according to their abilities.

3.4.2 Attitude

Attitude is likely the most unique and significant notion in current psychology; it largely resides in the emotional realm and exerts tremendous control over behavior. According

to the traditional model of attitude (Pant, 2009), attitude has cognitive, emotional, and conative components. The cognitive component of a learner's behavior refers to how the learner perceives information about a certain topic (Mathematics). Affective relates to appraisal, feeling, and emotions. The conative indicates purpose or choice. The term attitude has not been fully separated from terms such as trait, opinion, disposition, interest, value, and temperament, and this distinction is likely impossible. A person who identifies a pleasant impact or emotion with a certain psychological item is said to have a favorable attitude toward that object or subject.

According to studies, a negative attitude toward mathematics stems mostly from the connection between students and teachers. Therefore, if instructors have good attitudes toward mathematics, these views may increase their own global self-concept while impacting and benefiting students' mathematics learning. However, those with a bad mathematical outlook are frequently found to have a negative self-image and a sense of inadequacy.

3.5 Summary

We have discussed in this chapter some problems of teaching and learning of mathematics in Nigeria. These includes; content issue, curriculum integration, problem of understanding, issues relating to instructional methods and materials, classroom organization and teachers' factor. Factors relating to learners such as individual difference and attitude were also discussed. Finally, some ways to resolving some of these problems were suggested.

Student Activity

1. Lack of qualified teachers, instructional materials and appropriate methods of teaching are recognized as the pedagogical problems facing the Nigerian Mathematics teachers. Discuss.
2. Outline three (3) implication of large class and suggest three (3) ways of tackling them.
3. Suggest any three (3) methods you will use to cater for individual differences in your mathematics class.
4. Mention any three (3) strategies you will use in order to develop positive attitude of your students/pupils in mathematics learning.

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CHAPTER FOUR

INNOVATIVE INSTRUCTIONAL STRATEGIES

4.1 Introduction

According to Ameen & Salman, (2016) and Khadija (2018) that some contents in recent curriculum in mathematics are found to be difficult to be taught by some mathematics teachers and also found to be difficult to understand by students. Students recorded failure in these contents both in internal and external examinations. Recent mathematics curricula deemed tough topics include, among others, bearing, differentiation and application, geometric construction, logical reasoning, integration and applications.

Concern has been raised about mathematics education in secondary schools in Nigeria owing to pupils' persistently poor performance, which is linked to weak pedagogical tactics employed by mathematics teachers (Kajuru & Popoola, 2010). It is quite regrettable that the traditional technique employed to teach mathematics in secondary schools is teacher-centered, i.e., the instructor performs most of the talking while the students listen and take notes. They emphasized that instructional method had less of an effect on student performance. However, it is necessary to seek out instructional methods that have a favorable effect on students' performance, which affects whether they are motivated and engaged in the learning process. The need for innovative strategies in teaching mathematics and is borne out by the facts that different situations which include diagnosis and remedy of difficulty concepts and teaching topics of upper basic Mathematics curriculum, skills intended to be acquired by the learners, demands for different teaching strategies among others. Among the innovative teaching strategies to be discussed are: discovery method, team teaching, instructional scaffolding, peer-tutoring, concept mapping and cooperative learning.

4.2 Objectives

By the end of this chapter, you should be able to:

i. Explain the innovative instructional strategies, such as scaffolding, problem solving, peer tutoring, team teaching, among others
ii. design a lesson model for each of the strategy
iii. use the lesson model in teaching mathematics

4.3 Innovative Teaching Strategies

Teaching methods have changed and demonstrated a distinct shift from a teacher-centered to a learner-centered orientation, and mathematics and students are, of course, crucial to the teaching process. NTI (2010) and Khadija (2018) have identified

instructional scaffolding, discovery method, cooperative learning, problem-solving strategy, team teaching, using teaching facilities and ICT as a strategy to improve teaching and learning of mathematics as innovative pedagogical methods for improving the teaching of mathematics and the contents perceived as difficult by students. These tactics are deemed learner-centered, interest-generating, and activity-based.

4.3.1 Instructional Scaffolding

Scaffolding is a teaching strategy in which the instructor mimics the intended learning task and progressively transfers responsibility to the pupils. The teaching method is characterized as one that focuses on incrementally increasing pupils' capacities and decreasing help as they improve. This promotes active student participation in the teaching-learning process (Ahmad, 2016). The technique is learner-centered and is applicable to any subject at any level. The scaffolding method is a major component of cognitive apprenticeship, in which students grow increasingly proficient as problem-solvers through coaching, task structure, and suggestions without being directly given the solution. The instructor aids the students in doing the learning task since the teacher owns the task's control and must:

§ Assist the student in obtaining the scientific abilities you desire to teach, which is the lesson's purpose;

§ Assessing the students' prior knowledge (i.e., ZAD) via discussion and brainstorming.

§ Developing activities: identifying the next step of what the students aspire to learn and accomplish (ZPD) by group discussion and individual work.

§ Closing activities: evaluating each student individually at the conclusion of the class to evaluate whether or not they have mastered the objective.

4.3.2 Collaborative Instructional Strategy

The phrases collaborative learning and cooperative learning are equivalent and are used interchangeably. According to Kajuru and Popoola (2010), the most successful method of teaching mathematics is a cooperative instructional technique that is activity-based. It supports academic objectives and good performance, and is particularly successful in enhancing pupils' cognitive accomplishment. It has become an acceptable alternative to the old paradigm due to its significant role in enhancing the academic and social engagement of all students. The following lesson approach has been seen to expose students to collaborative learning strategies, per Ahmad (2016):

§ Opening Activity: evaluating the previous lesson, reviewing homework problems, and presenting the new assignment.

§ Development Activity: dividing students into small groups and assigning them a task; debating the response with the entire class; calling on a specified number and group

of students to respond to a question.

§ Closing Activity: pupils are given homework to turn in the next class period.

4.3.3 Method of Concept Mapping

This is another method for enhancing mathematics instruction and learning. Concept mapping is a metacognitive method used to assess an individual's knowledge structure, and it is an instructional method that stimulates learners' deductive thinking. According to Khadija (2018), concept mapping is the process of creating maps or diagrams to show the links between instructional ideas. To facilitate comprehension, the concepts are organized, simplified, and grouped hierarchically from general to specialized. According to reports, the method improves critical thinking and facilitates recollection of previously taught content (Omoroh, Peace & Adiri, 2019). In addition, they emphasized that the technique is an effective learning tool for teaching mathematics since it clarifies, defines, and specifies concepts and their relationships. They emphasized the following approach for creating idea maps:

§ Record the concepts or keywords used during the lesson;

§ Arrange the concepts and main ideas in a hierarchy from general to specific;

§ Connect the concepts by arrows by linking words so that each branch of the map can be read from the top to the bottom;

§ Provide examples, if possible, at the end of each branch;

§ Cross-link hierarchies or branches where appropriate.

4.3.4 Peer Tutoring as an Approach

It is a method for diagnosing and resolving problematic mathematical concepts, and it enhances the teaching and learning of mathematics. Peer tutoring is a phrase that has been used to represent a variety of tutoring arrangements. The majority of research on peer tutoring identifies it as an instructional approach that pairs students to learn or practice an academic task. It is also the procedure between two or more students in a group in which one student acts as a tutor for the other pupils. Peer tutoring is defined by Etsu and Manko (2019) as an educational technique in which students assist one another in learning content, reinforcing abilities, or practicing a learnt activity.

There are two primary forms of peer tutoring: incidental peer tutoring (IPT) and scheduled peer tutoring (SPT) (SPT). IPT frequently takes place at school or at home while kids are playing or socializing and guiding others is considered Incidental Peer Tutoring (IPT). SPT the other hand refers to peer-tutoring implemented in specific and for specified academic tasks, following a well-defined plan created by the teacher. Multiple organized peers tutoring programs, including cross-age peer tutoring (CAPT), peer aided learning methods (PALS), same-age peer tutoring (SAPT), and class-wide

peer tutoring, have been found to be helpful in teaching mathematics and physics (CWPT).

4.3.5 Discovery Method

As its name implies, the discovery method is a very suited approach for teaching mathematics at the secondary level. It is a tactic in which students strive to recognize their own knowledge, facts, and concepts in front of the teacher (Agwagah, 2013). Brunner's idea of discovery learning is the theoretical foundation of a problem-solving technique for science subjects, including unknown mathematical abilities, concepts, or principles, and it promotes student engagement. Through the discovery of information (concepts and ideas) on their own, a student's motivation is rewarded and their recall capacity is enhanced. Discovery-based learning facilitates comprehension of the structure of information. When one comprehends the structure of a subject, one comprehends how it relates to other subjects.

4.3.6 Problem – Solving Strategy

According to Kajuru and Popoola (2010), issue solving is a talent that demands developing unique and new solutions to recognized problems. In addition, it is the capacity to adapt useful procedures from tasks only marginally linked to the current activity and to produce potential solutions for solving known difficulties. Students in the mathematics class are actively engaged and create passion and interest for the topic. Teachers and parents play key roles in the growth and development of problem-solving abilities in society's learners. This will help students to adopt cognitive strategies for solving issues with minimal or no aid from peers or professors, and it will alter the students' attitude towards information. They view knowledge as provisional as opposed to permanent. They obtain fresh information.

4.3.7 Team Teaching

Using team teaching increases teaching and learning of Mathematics as indicated by researchers. It is an instruction that includes two or more individuals to handle task. This is contingent upon the amount of instructors instructing the topic in the classroom. There are three phases to the adoption of team teaching. These include the lecture to the entire class, the group (stream) tutorial, and the individual study. It develops the attitude of collaboration among teachers and other personnel in the school. Students are exposed to outstanding teaching since they are taught by the “expert” instructors in the school. Each student is better able to identify his position in the crowd, respect it, learn from its behavior pattern, contribute to its unity, and experience

the influence of group dynamics as a result of the integration of several instructional streams.

4.3.8 Task Analysis Model as strategy

According to Opayinka, Kehinde, and Kadejo (2019), the approach is the process of methodically breaking down learning information into sub-units that range in complexity from simple to most complicated. The technique is crucial in mathematics, as well as in all other fields of study and in everyday life. In this technique, the teaching of a new concept requires the scientific instructor to be familiar with the idea's sub-units, so that students understand what they need to know and it is easier to illustrate how the activities go together. Therefore, it is crucial to evaluate all sub-units that contribute to the primary notion so that it may be implemented progressively. The technique allows the instructor to analyze the student's abilities, the processes required, the curriculum's objectives, and how to assist the students' learning outcomes.

4.3.9 Programmed / computer- Assisted Instruction

It is a self-instructional method of education that enhances teaching and learning. The type of instruction that follows a sequence. This indicates that the learning information is delivered in an organized, structured, and organized manner prior to the learners beginning the activity. It is characterized by the fragmentation of learning knowledge into little pieces that progress a pupil from familiar to unfamiliar and basic to complicated. The fragments of information are given sequentially in phases. Whenever a response is correct, reinforcement gives by the immediate confirmation of the right answer or a correction of the wrong answers. The operation of programmed instruction is facilitated by two primary materials or devices. These are programmed textbook and teaching machines.

A programmed textbook is a self-instructional textbook; it is a written programme in a subject field where the subject content has been broken down into discrete learning sequence components. The programme is written as a series of easily answered questions that lead the student to logical conclusions for seen by the programmer.

The teaching machines are devices for self-instructional materials. The efficacy of the machine is dependent on the components employed. A well-designed machine presents one frame at a time, and the student controlling it brings each picture into view as required. Through programmed instruction, instruction is customized. Having individual instructors for each student is essentially equivalent. Individual variations in learning are addressed, and each student works at his or her own speed. Consequently, the learner's motivation to cover more frames is unquestionably influenced by the immediate knowledge of the outcome.

Student Activity

1. Write briefly on the following strategies:
   a. Scaffolding b. Team teaching c. Problem solving
   2. Enumerate the types of peer-tutoring
   3. Outline the features of lesson model in cooperative learning
   4. Distinguish between team teaching and peer-tutoring

4.4 Summary

A notable characteristic of a good mathematics is willingness to learn and accept rational innovations. Mathematics teachers must recognize his/her strength and weakness and also ready to learn for innovation in teaching of mathematics. In this chapter innovative teaching of perceived difficult concepts in mathematics were enumerated and explained, to make teaching effective and easy to the learners.

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CHAPTER FIVE

MATHEMATICS WORKING TOOLS

5.1 Objectives

By the end of this chapter, you should be able:

i. define and explain classroom management
ii. explain the importance of classroom management and mention at least major components of classroom management
iii. explain the meaning of discipline
iv. identify qualities of disciplined students and teacher
v. define and explain the followings: mathematics syllabus, scheme of work, unit plan, daily plan
vi. Outlines the features of unit and daily plan
vii. Explain the importance of lesson plan in mathematics teaching

5.2 Introduction

Mathematics classroom management is the process of producing a conducive learning environment in the mathematics classroom, whereas classroom discipline is the amount to which students are self-disciplined and eager to apply themselves to the tasks assigned by the instructor. In this chapter, working tools include: syllabus, scheme of work, unit plan and lesson plan were explained.

5.3 Classroom Management

The process of comprehending, sustaining, inspiring, and regulating the people and material resources in the classroom for optimal achievement in any teaching-learning environment is referred to as classroom management or classroom administration. You must use the resources at your disposal effectively to accomplish the goals mentioned. There are various strategies of managing mathematics classroom, some of which includes:

- Physical arrangement
- Chalkboard arrangement
- Class control
- Time management
- Sustenance of learners' interest
- Teachers' personality
- Managing behavior problem

5.4 Importance of Classroom Managements

§ It enables the teacher to choose an adequate topic to teach and set a correct objective for the lesson

§ It helps the teacher to choose an adequate disciplinary measure and utilize this effectively in class.

§ It enables him to prepare well for is lesson by choosing correct instructional material and having a good master of his subject matter in order to have the class under his control.

§ It enables him to maintain a system of records that provides easy asses to information needed for making managerial discussions.

§ It leads to teacher-student and student-student relationship which will eventually leads to effective teaching- learning situation.

5.5 Disciplines in the Class Room

Discipline is the training of the mind and character of learners so as to make them obedient to the authority and developed self-control. Classroom discipline is the instruction that instills in students the habits of self-restraint, orderliness, good behavior, collaboration, and getting the most out of oneself. The tactics a teacher employs to regulate student actions and attitudes during instructional time constitute classroom discipline. A teacher who employs consistent discipline tactics manages the classroom more effectively than one who does not. Indiscipline in the other hand is the act of deviating from the accepted rules and norms of the class or society such deviation may be in form of lying, stealing, sex offence, disobedience, truancy, assault, insult, smoking, taking hard drugs and mass demonstration. Causes of indiscipline include: favoritism; rules not enforce; communication gap, lack of good leadership; teachers' unpreparedness; bad habit.

5.6 Effective Classroom Discipline

For effective classroom discipline, the following can be helpful:

1. Maintain your dignity: Be a good example or model. Do not get too popular with students. Don't lower your dignity by getting into unnecessary argument, confrontation, humiliation and blanket punishment with students/pupils.

2. Be consistent: Set acceptable standards for work. Set high standards of behavior and apply rules firmly and fairly. Do not relax on your principles. Show no favoritism in dealing with misbehavior. Be just and fair. Do not take action over a student if you have lost your temper. Give respect and accept respect. Be sympathetic with the students.

3. Be resolute in your decision-making in order to maintain your power in the classroom. Accomplish not tell kids what you want them to do in an apologetic manner; instead, use authoritative tones that indicate you demand obedience. Never issue a threat that you are unwilling to follow out. Judge only when all relevant information is revealed.

4. Use punishment and reward judiciously: punishment should attempt to halt or remedy inappropriate behavior. Rewards should be utilized to recognize and reward achievement and effort. Reward can function as an incentive. Punishment should never be so light that it becomes absurd, and rewards should never be so simple to obtain that they lose their value and worth. Punishment should always be proportional to both the offense and the perpetrator.

5. Be self-critical: Let the students see you as you are, that is, be yourself to avoid being embarrassed. To command authority, teacher must be aware of his/her weakness and strength. In the classroom, he should arrive before the class and begin on time, be prepared for the lesson, keep everyone occupied and interested, motivated all students, mark all work promptly and constructively encourage student contribution and try to achieve a working harmony.

5.7 Mathematics Syllabus

The mathematics curriculum is the outline of the mathematical principles to be studied. It is organized or structured according to the level or year of education in an effort to improve the teaching and learning process.

5.8 Mathematics Scheme of Work

A mathematics plan of work is a weekly grouping of mathematics curriculum subjects. This is accomplished by separating the mathematics curriculum into three sections, one for each term of the school year. Thus, mathematics scheme of work comprises of a goal and an itemized overview of the mathematics instructor to re-examine the number of activities that needed to be covered (i.e., prior knowledge) and make some reference to this mathematics scheme of work. In constructing a mathematics program of work from a mathematical curriculum, the following considerations must be made:

- The necessity for logical sequencing
- The age, aptitude, and prior knowledge of the students
- The amount of time necessary for each topic
- The number of effective weeks of learning in a term or year
- The number of periods each week
- The need for materials and resources

5.8.1 Characteristics of Effective Mathematics Work Plan

A quality mathematics lesson plan must have the following elements:

- § Periods
- § Topics
- § Subtopics
- § Instructional Materials
- § Instructional Objectives
- § Content/Teacher Activities
- § Student Activities
- § Evaluation
- § Bibliography

5.8.2 Advantages of Mathematics Work Plan

The following are some advantages or benefits of plotting:

§ A solid plan of work drawn out at the beginning of the term is a helpful guide for mathematics teachers to know how much material has to be covered and how much time they will need to complete it.

§ It aids the mathematics teacher in allocating time for each topic item and in planning assessment in advance.

§ The mathematics teacher utilizes the scheme of work to identify relevant learning experiences with appropriate examples.

§ Mathematics scheme of work enables school administrators and mathematics teachers to meet the special needs of students;

§ It helps to ensure the continuity and coherence of the learning process;

§ Mathematics scheme of work is a good resource for instructional guides but not masters in mathematics teaching

5.9 Lesson Plan for Mathematics Teaching

Notably, it is crucial to plan ahead of time so that the mathematics instructor is well-versed in the material and aware of any potential obstacles that may occur, so that the appropriate answers are available before the lessons are presented. In mathematics education, the lesson plan is the final stage of curriculum implementation and a detailed study of how a task will be taught throughout the session. It is intended to assist teachers in delivering successful and efficient math's lessons. Typically, there are two types of instructional plans: unit plans and daily plans.

5.9.1 Unit Lesson Plan

A unit plan is a plan designed to cover the work on a given topic for two or three days, a week, or more per week according to the mathematical scheme of work. One, two, or three weeks are always stated for the work to be completed in a particular mathematical area. The purpose of a unit plan is to organize the entire topic, regardless of how long it may take to teach. The unit's objectives, content to be taught, teaching technique, and assessment content are outlined in the unit plan. The unit plan contains the principal components. These include:

i. Subjects

ii. Goals

iii. Instructional materials

iv. Basic knowledge vs content, i.e., the number of lessons to be taught and the topic of each lesson, should be clearly defined.

instructional strategy

vii. Final assessment

viii. References

Unit Scheme Model Lesson Class: Lower Basic V

Measurements of Central Tendency

Objective: Students must: i. Sort the grades in ascending and descending order

ii. Compile scores into a frequency distribution table

iii. Compute for median, mode and mean

Teaching Supplies: chalkboard, ruler, student score sheets

Essential Knowledge: counting the digits 1 through 100

Content: Four lessons: Mode, Median, and Mean

Introduction to ascending and descending order

Lesson 2: Frequency distribution table drawing

3. Determine the median and mode of a score distribution

Lesson 4: mean calculation

Method: the related ideas were clearly taught using examples. The method is repeated. Students were given classwork and assignments for evaluation.

References: Mathematics fundamentals for Nigerian Schools Printed by MAN

5.9.2 Lesson Plan Daily

A lesson plan is a teacher's planned and arranged learning activities centered on a certain topic, for a specific set of students in a class, and for a thirty-five, forty-five, or one-hour lesson time. Lesson plan is a clearly, methodically, and orderly planned procedure of teaching technique, teaching facts, and teaching materials that a teacher has prepared for use in delivering a lesson in order to reach the lesson's projected purpose. The objective

of the lesson plan is to allow the instructor move rationally without being restricted by his or her notes. A lesson plan should not restrict the teacher's approach and should be flexible enough to accommodate unforeseen circumstances that may happen during the class.

5.9.3 Elements of Everyday Lesson Plan

The format of the lesson play may vary, but it should generally include the following information: Topic/Sub-topic, Class, Duration, Date, Specific Behavioral Objectives, Entry/Entering Behavior, Introduction, Instructional Material/Resources, Presentation/Development of the Content/Instructional Procedures, Evaluation, and Conclusion.

This relates to the component of the subject that the instructor want to teach, especially within a specific time frame. In the absence of a clear topic, the instructor is doomed to fail. Likewise, it should be neither too wide nor too narrow to be covered within the lesson's allowed time. Occasionally, it may be essential to further dissect an issue. Therefore, sub-topics are necessary to guarantee that the instructor's attention is focused. For instance;

Types of fractions or addition and subtraction of fractions are the subject of Fraction, Sub.

Class: Specify the Class clearly. Example Grade 5 or Grade II

Duration: This indicates how much time the instructor has to complete a certain lesson. The instructor should prepare his or her lessons so that they may be completed properly within the allotted time.

Date: This specifies the date on which the teacher intended to teach the lesson. It allows the instructor to enter accurately in the record book what he or she has taught. Example: October 15, 2022

Specific Behavioural Goal

It outlines the conduct that the instructor anticipates his pupils or students to display following a lecture. In other words, specified or performance objectives are statements that indicate what the learner should be able to do or achieve upon completion of a particular learning endeavor. It is essential to establish behavioral objectives in terms that can be measured and observed. The instructor should avoid using overly vague terms such as "to know," "to grasp," and "to value" when setting objectives. However, words and phrases such as "to write", "to draw", "compute, solve, discriminate, differentiate, list, and state" can be used to express particular behavioral/instructional goals. The behavioral objective statement influences the instructor's decisions on content, approach, and evaluation.

Entry and Entry Behaviors

This is a component that several educators discover perplexing. Many instructors, particularly student-instructors, believed it to be any prior learning. It is not unusual for a teacher to reference the last lesson he or she taught as prior knowledge or entry behavior, despite the fact that the lesson has no relevance to the current topic/lesson. However, entrance behavior refers to the prior information the instructor assumes the pupils/students have that is relevant to the current topic/lesson and upon which the new lesson might build. It should be highlighted that anything the instructor believes the students already know must be validated during the lesson's introduction.

Introduction

This section of the lesson plan is as essential as describing the behavioral objectives. One may say that the success or failure of every instruction rests heavily on what occurs here. This is when the instructor piques the students' interest in the lesson's subject matter. There are several methods in which a teacher might do this. One approach to achieve this is by questioning them on their expected entering behavior. Others include describing the lesson's objective(s) to the class; the teacher explains precisely what the pupils will be able to perform upon completion of training. It is also possible to do so by giving a brief but engaging tale that connects to the issue at hand, or by simply elevating an object or piece of equipment for them to explain. Or any other method known to put kids at ease before to takeoff.

Pedagogical Materials/Resources

These items are brought into the classroom by the teacher to enhance the students' understanding of his lesson. Those things aid teaching and learning and concretize abstract concepts. Each subject lends itself to the utilization of particular sorts of content. Material resources include graphs, actual items, and real or created graphics on cardboard paper. It should be highlighted that instructional materials alone do not make a lesson relevant or simple to learn; they are only beneficial to the extent that they are appropriate and are handled creatively by the instructor.

Presentation

Presentation is the core of a lesson plan, which consists of learning activities and resources, as well as the "how" and "what" for achieving set objectives. In other words, the presentation depicts the activities of the teacher, the students, and other information. Typically, presentation or instructional procedures are structured as a series of phases. Each phase describes the teacher's and students'/pupils' responsibilities.

Evaluation

This is another lesson plan step. The instructor tests or evaluates the pupils' understanding of the material. He may pose questions to the students. Whatever he does in terms of evaluation is intended to determine whether or not the students can demonstrate mastery of the lesson's essential component(s). The outcome will indicate whether the teacher should re-teach the lesson or correct the students' improper learning. Evaluation questions must pertain to the material presented and be clearly articulated.

Conclusion

This is an additional phase of the lesson plan that many teachers, particularly student-teachers or novices, find confusing. The conclusion should be a task that may be completed in class or at home. Questions in the conclusion should be more difficult than those in the evaluation.

The stronger the structure of a class and the specificity of its objectives, the better the rate of student success. Whoever fails to plan has failed to plan.

Daily Plan Lesson Model

Date: 15th August 2022
Name of Teacher: Aliyu Garba
School: Nasafaru Primary School
Class: Lower Basic One (1)
Number in Roll: 45
Sex: Mixed
Average Age 6 years
Subject: Mathematics
Topic: Whole Number (1-5)
Time: 08:00am – 08:30am
Period: 1st
Duration: 35 minutes
Instructional Material: Sticks, bottle tops, tins, pebbles, counting bars, etc.
Behavioral Objectives: By the end of the lesson, the pupils should be able to:

i. Association numbers with objects
ii. They should also be able to show the joy of using the numbers with objects.

Introduction (Entry Behavior): The teacher relates the previous express and /or knowledge with the present lesson e.g.

i. How many chairs do you have?

ii. How many eyes do you have? and so on

Presentation

Step I: The teacher presents the lesson by showing charts carrying number of different objects e.g.

The pupils should be asked about what they must have seen e.g. 1, 2, 3 etc.

Step II: The children should be given different objects to group and count e.g. = 1 = 2 = 3.

More activity on counting and grouping.

Evaluation: The lesson should be evaluated by asking the children questions based on the topic e.g.

i. identify the number of each object according to the group e.g.

= 1

= 2

= 3

= 4

= 5

ii. Count and identify numbers from 1-5

iii. Arrange objects according to numbers e.g.

3 = 4 = 5 =

Conclusion: The teacher summarizes the lesson for the children and finally gives them work & assignment e.g.

1. Bring 2 tins from your home
2. You bring 4 sticks
3. Bring 5 bottles tops tomorrow

5.10 Importance of Lesson Plan

§ A teacher who plans his lesson is less likely to waste class-time and is more likely to succeed where others fail;

§ Lesson planning enables the teacher to give consideration to such factors as the nature of the topic to be taught, the best way to put it across, and the possible problems learner may have learning it;

§ Lesson planning also gives the teacher confidence in front of his class and enhances a logical and sequential presentation of learning experiences;

§ • It helps the teacher streamline the content of his teaching so that it is neither too much nor too little for the available time;

§ It helps him prepare ready-made questions that direct the students to the correct objectives;

§ It helps him identify the teaching aids required for teaching the lesson.

§ It helps avoid the emergence of significant instances of disobedience.

Student Activity

1. Differentiate between scheme of work and syllabus
2. What is classroom management
3. Distinguish between classroom management and classroom discipline
4. Enumerate qualities of a disciplined students
5. Outline the importance of classroom management
6. Mention six major areas of classroom management
7. What is unit lesson plan
8. Mention and explain the major component of daily lesson plan
9. Advance five reasons why a mathematics needs to draw a scheme of work
10. Write behavioral objectives for the topics

a. Recognition of numbers in Nursery two
b. Ratio
c. Profit and Loss
d. Mean
e. Variation

5.11 Summary

This chapter covered detailed discussion of classroom management and classroom discipline. It showed that for the mathematics teacher to management his classroom well, he/she needs to manage his chalk board, sustain students' interest, and needs to communicate well, watch his personality. Discipline is discussed as the extents to learner areself-controlledand willingness applies themselves to their tasks. Syllabus and scheme of work were explained as working tools, including lesson plan.

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Niger State College of Education (2010). A handbook in mathematics for tertiary institution

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CHAPTER SIX

MAHEMATICS LABORATORY

6.1 Objectives

By the end of this chapter, you should be able to:

i. define mathematics laboratory
ii. state the functions of mathematics laboratory
iii. enumerate facilities in mathematics laboratory
iv. state some mathematics topics in mathematics laboratory
v. design and general layout of mathematics laboratory
vi. perform activities of mathematics laboratory lesson

6.2 Introduction

The wrong perceptions about mathematics by most secondary school students in particular and the public in general that mathematics is difficult, needs to be thrashed. This is attributed to abstract nature of the subject, among others. Practical tasks in mathematics will be conducted in mathematics laboratory to enable you achieve optimum potential through the use of the laboratory facilities. In this chapter will deals with following sub-units: concept of mathematics laboratory, facilities needed in laboratory, laboratory lessons and its functions in mathematics lessons.

6.3 Meaning of Mathematics Laboratory

The mathematics laboratory is a space where teachers and students may experiment and investigate mathematical patterns and concepts. There is a collection of games, puzzles, and other teaching and learning resources in this area. At other terms, a mathematics laboratory is a space in a school that has the necessary equipment for the practical instruction and study of mathematics. It gives the necessary experiences for the acquisition of mathematical ideas, principles, and generalizations. It is a resource centre designed for the growth of mathematical activities in schools that students and teachers utilize to investigate the world of mathematics.

6.4 Designs and Basic Format

Since mathematics lab activities need students or teachers to explore the world of mathematics in order to learn, discover, and create an interest in the topic, you are persuaded that it is essential in your school. These are recommended laboratory design and overall layout:

6.4.1 Room Accommodation

The recommended design and basic arrangement of the laboratory should provide seating for around 30 students at a time. Ensure the cleanliness of the laboratory and its equipment, and offer at least two escape doors.

6.4.2 Equipment and Materials

The minimal materials required to be retained in the laboratory may comprise all important equipment, raw materials, and other items necessary to properly carry out the tasks. The number of various items may vary from school to school based on the size of the class. Sharp objects should not be carelessly left out in the open, as stock records are crucial to the efficient running of a laboratory. Some of the necessary resources are provided in the subsequent unit.

6.4.3 Human Resources

It is preferred that a person with a Bachelor of Education (Mathematics) or above be placed in charge of the mathematics laboratory. He/she is required to have specialized knowledge and an interest in the subject in order to perform practical work. A laboratory attendant or laboratory assistant with the required qualifications and subject-matter expertise is an asset. The teacher and laboratory assistant must closely monitor the pupils' activities to verify that their behaviour conforms to laboratory norms and regulations.

6.4.4 Allocation of time for activities

15 to 20 percent of the total available time for mathematics should be allocated to activities. In the timetable, slots for laboratory tasks can be allocated appropriately.

6.5 Infrastructure at the Mathematics Laboratory

The purpose of the mathematics laboratory is to make learning mathematics enjoyable and engaging, and to encourage individual involvement in such activities. The sort of experiment required is determined by the materials necessary for the goal. Stones, bottle tops, sticks, beads, empty tins, graph papers, duplicating sheets, cardboard, sheets, rubber beads, instruments in mathematics sets, clocks, iron rod, cello tape, plaster, wire point and paint brush, sand, glue, number charts, balance scale, thermometer, beakers, measuring cylinder, screw driver, knives, models, pliers, nails, razor blades, shears, magnetic compass, geo-board, hammer.

These materials are obtained within the environment of the teacher and student. They are always available and not too expensive, so the materials are group into two. Improvise and purchase. Some of laboratory instruments used in determining some concepts are as follow:

Length – Scale ruler, tape rule

Mass – spring balance

Volume – cylinder, beakers, burette, pipettes, volumetric flasks and conceal

Weight – weighting scale or weighting balance

Temperature – thermometer

Time – clock, stop watch

Mass and weight are frequently used interchangeably. The two names, however, have different connotations. The following table shows the differences between the two values:

S/N	MASS	WEIGHT
1.	the amount of material in a thing	the gravitational force acting on an object
2..	From one location to the next, the object's mass is constant	The area affects the weight of items
3.	Grams are used to measure mass	Weight is measured in Newton

Activity 1

1. What is mathematics laboratory
2. Give five activities that you can be used in mathematics laboratory
3. Give 10 materials that can be used in mathematics laboratory
4. Explain the design and general layout in mathematics laboratory

6.6 Functions of Mathematics Laboratory

Some of the ways in which activities in mathematics lab could contribute to mathematics learning of the subject are:

i. It assists in taking care of individual differences, and enrich students with adequate mathematical skills

ii. It is a resource centre because it serves as a research centre which could lead the teacher and student to discover or verify some mathematical laws, formulae and theorems.

iii. Collaborative work in a mathematical laboratory usually encourages sharing of knowledge and cross breeding of ideas

iv. Mathematics laboratory enables the students to know and familiarize themselves with names of the equipment kept there and their functions.

v. Mathematics laboratory helps to eliminate or minimize the abstract nature of mathematics concepts since concrete or visual materials are used to teach/learn such concepts.

vi. It assists in creating motivation and building positive attitude, confidence and reducing anxiety towards mathematics.

vii. It helps the students to relate the mathematical concept learnt to real life situation through various practical activities.

viii. An activity involves both the mind and hands of the student working together which facilitates cognition.

ix. It provides opportunity to students to repeat an activity several times. They can revisit and rethink on a problem and its solution.

6.7 Types of Laboratory lessons

Lessons in mathematics laboratory solely depend on the objective of the individual carrying out the experiment. Below are some of the stated lessons that can take place in mathematical laboratory.

Numbers and Operations

§ Use of charts to teach number pattern such as even, odd, prime number, multiplication factors multiple

§ Use of counters to teach counting and basic operation

§ Use of abacus place values

§ Materials in shopping corner in the laboratory for teaching buying/selling

§ Verify the result of the product of two numbers of the same signs and those of opposite signs

§ Compute Pascal's triangle for the coefficient of binomial expansion of power 20.

Statistics and Probability

§ Tossing a coin for prediction purpose and finding the probability of an event

§ Collect grades in two subjects and find their coefficient of correlation.

§ Collect original data from a survey to determine what variable are related.

Ratio and Proportion

§ Compare the ratios of a sizes of similar triangles and the ratio of their areas

§ Compare the volumes of a cube to the one whose size is of length five times the first by pouring and deducing the ratio of their volume

Geometry

§ To carry out paper folding activities to find;

- The midpoint of line segment
- The perpendicular bisector of a line segment
- The bisector of an angle

- The perpendicular to a line from a point given outside
- The perpendicular to a line from a point on the line

§ To carry out activities using geo board to;
- Find the area of triangle
- Find area of any polygon by completing the rectangle
- Obtain a square on a given line segment
- Given an area, obtain different polygons of the same area

§ Calculate the angles of elevation of an object on top of a tree using clinometers
§ Calculating the distance between two points on the parallel of attitude
§ Sketching different types of curves
§ Measurement of
- Length
- Volume/capacity
- Weight/mass
- Time
- Temperature
- Angles

§ Using materials to drive the formula for
- Perimeter of polygons (rectangles and triangles)
- Areas of rectangles/square and triangle
- Circumference and area of a circle
- Constant **π**

Trigonometry

§ Obtain a trigonometrically ratio table of sines, cosines and tangent of angles 30, 60 and 45 from equilateral and isosceles triangles

Deduce by cutting right angled triangles of sides in multiple of 3, 4 and 5 that a triangle of side 3n and 5n is a right-angled triangle where n is a natural number.

6.8 Preparing for a laboratory lesson

Laboratory lesson refers to the laboratory method of instruction. This approach requires a laboratory equipped with mathematics-related equipment and other important teaching tools. For instance, equipment pertaining to geometry, trigonometry, mathematical models, charts, balances, different wooden or hardboard figures and forms, graph paper, etc.

This approach is founded on the adage "learning by doing." It is a strategy that helps pupils to discover mathematical facts via activities. It proceeds from the tangible to the abstract. Laboratory approach is a technique that stimulates students' actions and motivates them to create discoveries.

The following approach must be followed by teachers while organizing mathematics laboratory lessons.

§ Aim of the practical work: the teacher clearly states the aims of the practical work or experiment to be carry out by the students.

§ Provided materials and instruments: the students are provided with the necessary materials and instruments.

§ Provide clear instructions: that is clear procedure of the experiment.

§ Draw the conclusions: The students are required to draw the conclusions as per the aim of the experiment.

Example 1

Derivation of the formula for the volume of a cone

Aims: To derive the formula for the volume for the volume of a cone

Material and Instruments: cone and cylinder of the same diameter and height at least 3 sets of varying dimensions, sawdust, water and sand.

Procedure: students to do the following activity.

i. Consider each pair of cones and cylinders that are the same height and diameter.

ii. Record the height and diameter.

iii. Count how many times the cone is dumped into the cylinder and record the number in a table column.

iv. Sand, water, or sawdust should be poured into the cone until it is completely full.

v. Convert the other two sets of cones and cylinders to use in the experiment, and record the reading as before.

S/N	DIAMETER OF CONE/CYLINDER	HEIGHT IF CONE/ CYLINDER	NO OF MEASURE OF CONE TO FILL THE CYLINDER
1	3cm	6cm	3
2	5cm	7cm	3
3	6cm	10cm	3

Drawing Conclusions

Each time, irrespective of the variations in diameter and height it takes 3 measures of cone to fill the cylinder.

Volume of cone = 1/3 volume of cylinder

But volume of cylinder = $\pi r^2 h$.

Volume of cone = 1/3 $\pi r^2 h$.

Example 2

Sum of three angles of a triangle is 180 degrees. How do we prove this in the laboratory?

Aims: To prove that sum of the three angles, of a triangle is equal to two right angles or 180 degrees

Materials and instruments: Cardboard sheet, pencil scale, triangle, scissors, razor blade.

Procedure: In the laboratory, students will be given cardboard sheet each and then they are told how to draw triangle and cut out separately with the help of scissors.

Observations: Students will measure the angles of the triangles drawn and write these in a tabular form.

FIGURE NO	MEASURE OF DIFFERENCE ANGLES			ANGLES
1	A	B	C	A+B+C
1	90	60	30	180
2	120	30	30	180
3	60	60	60	180

Calculation: after calculating the angles of several triangles represented on cardboard sheets. We compute their total and draw a conclusion. Students will be able to draw the inductive conclusion that the sum of a triangle's three angles is 180 degrees by computing the triangle's three angles in this manner.

Example 3

Topic: Congruent Triangles

Objective: At the end of this lesson, students should be able to:

Identify the properties congruent triangles

Materials: from paper cut triangles of the same size and different sizes, using ruler and protractor

Procedure: The significance of triangular congruency is explained to the pupils by the teacher.

The term "congruent" refers to two or more triangles that are identical in every way. When one is pulled up, it may be put perfectly atop the other since their sides and angles are both equal. additionally have equal areas. Below are the conditions

1. The three side are equal to another triangle, represented by side, side, side (SSS)
2. Two sides are equal to two side of another triangle and the included angle, represented by (SAS)
3. Two angles of are triangle equal to another triangle and any one side of the triangles, represented by (AAS).

Step II: Teacher provides much paper cutting of triangles and call on some students to identify pairs of congruent triangles. The students should do this by actually placing one triangle out on another until they get those that fit into one another without any left over. After a series of such exercise, you should ask them to compare:

i. The length of corresponding sides by actual measuring ruler and

ii. The size of corresponding angles by using a protractor.

Step III: Teacher should demonstrate with the paper cutting and actual measurements to establish the condition for congruent of triangles.

Example 4

Topic: Similar Triangles

Objective: At the conclusion of this lesson, students should be able to recognize triangles that are similar.

Triangles are said to be similar, or to have the same shapes, when their respective angles are equivalent to those of other triangles. This doesn't mean that their sides are congruent or equal; rather, it just means that the ratios of each side are the same. For e.g s ABC and XYZ are similar then A = X, B = Y, C = Z

Also the ratio of the corresponding sides is equal.

i.e AB = AC = BC. Similar s is provided with right angled triangle whose lengths are (3,4,5),

XY XZ XY

(9, 12, 15) and (5, 12, 13), (10, 24, 26) the students should identify them as similar triangles by comparing their angles. Let students calculate their areas. For the (5, 12, 13) then area in ½.5.12 = 30 sq units while that of (10, 24, 26) is ½.10.24 = 120 sq units. The formula 1/2bl is used because the s are right angle s. the ratio of pairs of corresponding sides is ½, the square of ratio of the corresponding sides is ½, the square of this ¼. The ratio of the area is 30/120 = ¼, so ratio of areas = square of ratio of the corresponding sides. Consider the second pair of s use

have the ratio of area = ½.3.4 = 1/9,

the ratio of the corresponding sides 3/9 = 4/12 = 5/15 = 1/3

½.9.12

Square of ratio of corresponding side = 1/9.

By generalization the students will learn that if two triangles are similar, the ratio of their areas is equal to the square of the ratio of the length of corresponding sides. You may end the lab lesson by giving the summary of conditions of similar triangles that students should take note.

i. When two angles from one triangle are equivalent to two angles from another.

ii. If their matching sides' ratios are equal.

iii. If the ratio of the triangles' areas to their squares of matching sides is the same. Equiangular triangles are included in the category of similar triangles.

Example 5

Topic: Perimeter of plane shapes

Objective: The student should be able to obtain of the perimeters of plane shapes.

Material: Cardboard, thread, ruler

Let your students obtain the perimeters of rectangle, square parallelogram, quadrilaterals and triangles and irregular shapes.

The perimeter can be measured by string on irregular shape the surface while we use the string to trace the other edges till we get back to our starting point.

Perimeter of plane shapes, they can use ruler to measure each side directly and sum up the measurement obtained.

Examples: find the perimeters of the following shape whose dimensions are shown.

1. 2. 3.

Solution

1. Perimeter = 7+8+9 = 24 units
2. Per = 5+17+5+17 = 44 units
3. Perimeter = 5+7+3+6 = 21 units

6.9 Mathematics Laboratory Based Approach (MLBA)

This is learner centered activities carried out in the mathematics lab for teaching-learning, which include experimentation, exploration, inquiry, discussion, project work and cooperative learning. Students manipulate the materials in the learning process and the teacher plays the role of a facilitator. The approach has merits and demerits and itemized as follows:

Merits of MLBA

i. The approach is founded on the notion of learning by doing.
ii. It is based on the student's self-pacing
iii. It clarifies key concepts, principles, etc.
iv. It fosters self-confidence and teaches pupils the value of hard work.
v. The students are instructed in the usage of various laboratory equipment.
vi. It helps pupils acquire a habit of scientific inquiry and research
vii. This approach introduces mathematics as an applied topic viii.
viii. It affords pupils opportunities for social contact and teamwork.
ix. It is student-centered and engages pupils since they actively participate in the learning process.

Demerits

i. This method can be used for a small class only
ii. It requires a lot of planning and organization
iii. This method it is not possible to make progress quickly.
iv. This method it is suitable only for certain topics.
v. This method requires laboratory equipped with different apparatus
vi. Not all teachers can use this method effectively.
vii. It is an expensive method because not all schools can adopt the method
viii. This method has very little of theoretical part in it.

Activity 2

1. Enumerate five functions of mathematics laboratory
2. Give five activities that you can be used in mathematics laboratory
3. Give 10 materials that can be used in mathematics laboratory
4. Explain five (5) merits of mathematics laboratory-based approach
5. List instructional materials, used in measuring

a. Length b. Volume/Capacity c. Weight/Mass d. Temperature e. Time

6. Choose any of the topic and prepare a mathematics laboratory (practical lessons) to be carried out in a named class
7. Give two activities of laboratory lesson on statistics and probability.
8. State three functions of a mathematics laboratory.
9. Prepare a mathematical laboratory lesson for finding the curved surface area of a cylinder in terms of a rectangle.
10. Draw and list the properties of the following solid shapes (i) Cuboids (ii) Cone (iii) Cylinder (iv) Tetrahedron
11. Given a cardboard sheet, cello tape and a pair of scissors, explain how you would make (i) a cube (ii) square based pyramid
12. What instructional materials will you see in the teaching of the concept of similarity as applied to triangles in mathematical laboratory lesson?
13. Discuss the use of graph paper in the teaching of similar triangles.
14. Why is it necessary to use instructional materials in teaching congruence of triangles?
15. What is significance of the order of the labeling two congruent
16. List the three test for the congruency of s

6.10 Summary

§ Mathematics laboratory is a room set aside in school build where materials/equipment are kept for teaching and learning mathematics in a practical manner

§ Activities in mathematics laboratory are carryout either individual or group

§ The functions of mathematics laboratory are aimed at making teaching-learning of mathematics interesting, joyful, confidence and permanent in mind of learners and reduce mathematics- anxiety. It encourages students to become autonomous learners and allows individual students to learn at his or her space.

§ Design and general layout provide functional mathematics lab, both materialand human resources

§ Some laboratory lessons that can be presented through MLBA, which include: area of plane shapes using geo-board, volume of figures, tossing of dices, making of models and charts.

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CHAPTER SEVEN

GEOMETRICAL CONTRUCTIONS

7.1 Objectives

By the end of this chapter, you should able to:

i. identify mathematical instruments and their uses
ii. differentiate between draw and construct in geometry
iii. bisect any given line or angle
iv. construct special angles
v. construct a triangle or any shape
vi. define the locus of a point

7.2 Introduction

This chapter is dedicated to simple construction. Looking at structure and objects around us there is evidence of geometrical constructions and that make mathematics real to life. Laboratory activities involving geometrical constructs are provided for pupils. By the time students have acquired the abilities and mastered the lessons, mathematics will be viewed as a living, non-formula-based topic. In geometry, it is essential that pupils recognize the distinction between draw and build. In geometry, just a ruler and a pair of compasses are used to "build," however in "draw," any tool deemed required for producing such a drawing may be used. In this chapter, we will discuss the building of bisectors of lines and angles, as well as the development of forms and loci.

7.3 Components of Mathematical Instruments

Mathematics instruments are sometimes known as geometric tools. They tools used in constructing different types of geometric shapes. The most instruments (tools) that are used within the field of geometry, including the ruler, divider, protractor, set squares and compass. The description and uses of the tools are discussed in details in next unit.

Ruler **Protractor** **Pencil**

Dividers **Compass** **Set Squares**

7:4 Uses of Mathematical Instruments

Each of these tools is used in geometric constructions to provide psychomotor activities.

Ruler

It is also known as a straightedge or line gauge. It is a tool utilized in geometry and several engineering applications. A ruler is utilized for generating straight lines and measuring line segment lengths. It may also be used to measure the object's distance. One side is graduated in centimeters and millimeters, while the other side is graduated in inches. The space between the markings on the ruler is known as the Hash.

Compass

It is a V-shaped device that is characterized as a drafting tool with a sharp tip in the center and a pen or pencil at the other end. Adjustable is the distance between the pencil and the pointer. It is used to draw circles, arcs, and angles. In addition to marking equal lengths, it is one of the tools needed to construct a geometric circle.

Protractor

It is a semicircular disc that is used to measure and draw angles. It is graded from 0 to 180 degrees and can measure any angle within its range. 0 to 180 degrees are marked from left to right and vice versa. The protractor's inner and outer readings complement one another, and the total of the two readings equals 180 degrees.

When measuring an angle using a protractor, the following procedures must be observed:

§ if the angle to be measured is on the left side of the protractor, the outer reading is used; if the angle to be measured is on the right side of the protractor, the inner reading is used.

Divider

Divider resembles compasses in appearance because to its "V"-shaped construction. However, the "V" has pointers on both ends. The adjustable distance between them is used to measure and compare lengths. Frequently, the divider and compass are used to denote distance or division.

Set-Squares

The set-square is one of the mathematical tools typically seen in geometry boxes. These are the triangle plastic pieces with the space between them eliminated. There are two possible sets of set-squares. One angle is 45 degrees, while the other ranges from 30 to 60 degrees. The 45-degree set square has a 90-degree angle, while the 30-60-degree set square has a straight angle. The 45-degree set square is used to draw vertical lines, as well as parallel lines, perpendicular lines, and standard angles, among other things.

Student Activity 1

Draw out all instruments in mathematical set and their uses

7.5 Teaching of Construction

You must note the following when carry out construction. These are:

1. A very sharp pencil.
2. A very straight ruler.
3. A clear eraser.
4. A very good pair of compasses, so as to enable you arrives at a correct and neat construction.

1. To bisect a given line

To bisect a given straight line, a ruler and a set of compasses are required. In order to do the bisection, the following procedures must be taken

(a) Draw the given line AB

(b) With any convenient radius greater than half AB and center A, construct an arc above and below, the line

(c) With center B and same radius as above and below, draw arc to cut the previous ones at points C and D

(d) The line drawn through the points of intersections of the arcs will cut line AB at E

(e) Hence |AE|=|EB|

2. To bisect a given angle

In constructing a line called a bisector of an angle, you required a pair of compasses, a ruler, and a sharpened pencil. The following procedures are followed.

(a) Given angle XYZ

(b) Using Y as center, and any radius draw an arc to cut YZ at A and YX at B ii

(c) With center A and B and equal radii [either equal to AB or greater than half of AB], draw the area to meet at T. Join T

(d) Therefore XŶT= TŶZ.

3. **Construction of a perpendicular to a line**

In carrying out this kind of construction, there are two possibilities. The perpendicular can be from.

(a) A point X to the line

(b) A point X outside the line

The same construction instruments are needed as those in the preceding constructions.

(a) A perpendicular from a point X on the line.

Let the line be AB. With X as the center and a reasonable radius, draw an arc to cut AB at M and N. Then with M and N as the center and same radius, draw arcs to meet at Y. Then join XY, XY is the required perpendicular from the point XY to AB.

(b) A perpendicular from a point X outside the line.

Let the line be AB with X as centre and any convenient (or reasonable) radius, make an arc cutting the line AB at P and Q. then, with P and Q as centre and the same radius, make arc cutting each other at Y. then join XY. XY is the required perpendicular to the line AB

4. **Construction of an angle of 90^0**

Construction of angle of 90^0 is the same as a perpendicular line to a given line at a given point.

(a) Draw line AB\

(b) Mark any point C on AB

(c) Draw a semi-circle with center C and any convenient radius. Label it D and E.

(d) With center D and E at different times and same radius draw arcs to intersect one another at point F

(e) Join CF. note: FCB=FCA=90^0

5. Construction of angle of 45^0

(a) Construct angle BAC= 90^0

(b) Bisect angle 90^o to give angle 45^0

6. Construction of an angle of 60^0

(a) Draw a line AZ

(b) Mark a point X on AZ

(c) With center X and any convenient radius, draw an arc to cut AZ at D.

(d) With center D and the same radius as XD, draw an arc to cut the previous one at E

(e) Draw a line from X through E [line XY]. YXZ is the required angle 60^0

To construct an angle of 30^0

(a) Construct an angle of 60^0 (as indicated above)

(b) Bisect the angle of 60^0

(c) That gives the required angle 30^0.

7. Construction of an angle of 15^0

(a) Construct the 60^0angle

(b) Bisect the angle 60^0 to give 30^0 angle [as shown above]

(c) Bisect the 30^0 angle to give 15^0 angle

(d) ABC is the required 15^0 angle

8. Construction of multiple angles

Construction of angles like 60^0, 120^0 and 90^0 are directly constructed without any difficulties. But some special angles are done through adding or subtracting as the case may be. Such special angles are

(a) Construction of angle 105^0

Angle 105^0 is the sum of angle 90^0 and 15^0ie $90^0+15^0=105^0$.hence, angle 105^0 is constructed.

(b) Construction of angle 75^0

Angle 75^0 is the sum of angle 60^0an 15^0

(c) Construction of angle 150^0

Angle 150^0 is the sum of angle 120^0 and 30^0, or substruct 30^0 from 180^0

(d) Construction of angle 135^0

Angle 135^0 is the sum of 90^0 and 45^0 or 120^0 and 15^0.

Student Activity

1. Construct angle 60^0, 120^0 and 90^0
2. Draw a line AB, locate a point X on the line

With point X construct angle 135^0. Bisect the just constructed angle.

3. Draw a line |XY|=10cm. locate a point P such that |XP|=2cm locate another point Q on the line such that PQ=5cm. construct the perpendicular bisector of PQ.

9. Draw a parallel line.

(a) Draw the line XY. Make points 'a' and 'c' near to the ends of the line respectively and 'b' about the center of the line.

(b) With centre's a b and c and a radius equal to the distance away from the required parallel, draw arcs.

(c) Draw a line across the tops of the area. This is the required parallel line

10. Construction of a parallel line to a given straight line through a point

(a) Draw straight line XY and a point P which is not on the line

A parallel line can be constructed to line XY through point P thus:

(b) Mark any two points A and B on XY. With centre P radius equal to AB, draw an arcs S, with centre B and radius equal to AP. Draw another is to cut S at T.

(c) Draw a line to join P and T together. PT is the required parallel line to XY

11. Construction of an Equilateral Triangle.

This is based on the construction of an angle of 60^0 but the lines are joined to form the required triangle. An equilateral triangle is a triangle with all angles equal.

12. Construction of an Isosceles Triangle

An isosceles triangle is a D with two of its three sides equal and the two angles facing the equal side, equal too. Now let us look at how it illustrated construction.

(a) Draw line AB

(b) With A as centre and any reasonable radius not equal to AB, make an area of a circle.

(c) With B as centre and the same radius make another arc cutting the previous are at C.

(d) Join AC and CB. Hence, DABC is an isosceles D with AC=BC

13. Simple Constructions with Circle & Triangle

There are three simple constructions, which deal with triangles and circles. These constructions are.

In doing the construction suppose you have Triangle ABC on which a circum-circle is to be constructed:

a. Choose any two sides, say AB and AC

b. Construct the perpendicular bisector of the two sides AB and to meet at M.

c. Using M as the centre and radius equal to MB, draw a circle. The circle will pass through vertices A, B; C. this is the required circum-circle.

B. The inscribed circle [in circle] is a circle that is constructed inside an already triangle. The circle will touch the sides if the triangle sees the figure below:

The construction of an in-circle is done as follows:

Given a triangle XYZ to which an in circle is to be constructed

i. Choose any two angles of the triangle

ii. Construct the bisector of the two angles to meet at any point P.

iii. Construct a perpendicular from P to A on XY

iv. Then with centre P and radius PA, draw a circle.

The circle is the required in circle (in cribed circle)

The in circle is also called inscribed circle.

C. The E-scribed circle of a triangle ABC

It is the circle outside the triangle as shown in figure below with the three sides of the triangle as tangents to the circle. The circle is exterior to the triangle and called the ex-scribed circle.

14. **Locus of a point**.

The locus of a point is the path traced by that point under specific conditions. All point on that path satisfies the condition (S). The plural of locus is loci

Example of same common 2-dimensional loci

(i) The locus of a point which moves with a constant distance from a fixed point in a circle; with the fixed point as the centre and the constant distance as the radius. This is illustrated as below

(ii) If the point can move anywhere in space with the constant distance from a fixed-point O, then the locus is a sphere with O as the centre, and r, the constant distance as the radius of the sphere.

(iii) The locus of a point in a plane, which is always equidistant from fixed points is the perpendicular bisector of the line joining the two points see the figure below

Student Activity 2

1. Distinguish between construction and drawing

2. a. what is the locus of a point

a. Sketch the locus of a point in space moving with a constant distance from that fixed point. Name the product.

3. Construct a triangle PQR such that |PQ|= 7cm, |PR|= 6cm and |QR|= 8cm. construct the bisector of P meeting QR at X. Construct the bisector of PR meeting PX at Y. Measure |QY|

4. Describe how you would construct angle 120^0

7.6 Summary

In this chapter, you have learnt about the following.

- Mathematical instruments and their uses
- Differences between drawing and construction in geometry
- Construction of perpendicular line from a point X to the line and a point X outside the line
- Bisection of an angle
- Constructions of special angles 30^0 , 60^0 , 45^0 , 90^0
- Construction of triangles such as equilateral, scalene, etc
- Construction of parallel lines
- Locus of a point

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CHAPTER EIGHT

IMPROVISATION

8.1 Objectives

By the end of this chapter, you should be able to:

i. identify plane and solid figures
ii. name these figures
iii. enumerate the properties of some of these shapes
iv. modeling of these shapes
v. Define improvisation, learning aid, its classification, and its criteria for selecting learning aids.

8.2 Introduction

In this chapter, you will learn the learning aids and its criteria of selecting the aids. Also, discussion is made on various plane and solid shapes and their properties. Materials needed of modeling plane and solid shapes were discussed with practical.

8.3 Meaning of Improvisation

Improvisation is the use of alternate resources to enhance education in the absence or scarcity of a specific first-hand teaching tool. It is also the act of supplying instructional materials from our region when there is a scarcity or absence of conventional ones. It might be defined as the sourcing, selection, and deployment of relevant instructional aspects of teaching and learning in order to realize educational goals and objectives in a meaningful manner.

Instructional materials are the classroom resources utilized by mathematics teachers. These are materials utilized by both instructors and students to facilitate effective teaching and learning. It is utilized to enhance the teaching and learning of mathematics. They aid both the instructor and the student by concretizing what is taught and learned. These materials exist in various forms and may represent actual items. For example, maps, charts, photos, etc. These materials may appeal to the hearing, sight, or both senses.

Local resources that may be utilized to create mathematics teaching/learning aids are those that can be obtained from home or school contexts. These materials are readily available and inexpensive, although the necessary instruments can be rented from other sources. Materials include cardboard string, a ruler, plywood, nail drawing paper, graph paper, a thin iron rod, letter steeds, beads, gum, a blade, a knife, a saw, mathematics set tools, a hammer, etc.

8.4 What is learning aids?

The process of acquiring new understanding, knowledge, behaviors, skills, values, attitudes, and preferences is the definition of learning. It is any change in behavior that is relatively permanent and results from practices or experience. According to some mathematics educators, a learning aid in mathematics education is anything designed to improve mathematics learning and retention by learners. In a simple term it is any form of materials that can aid or speed up the process of learning mathematics with or without any assistance of a second person (mathematics teacher).

8.5 Classification of Learning aids

There are various classifications of learning aids in mathematics as listed below:

i. Traditional Learning Aids: This includes learning through books, periodicals, journals, projects, thesis, handbook, chalkboard, etc.

ii. Visual Learning Aids: VLA includes posters, models, figures, charts, graphs, etc. It also includes graphics, such as diagrams, cut-outs, globes, objects, cartons, bulletin boards, pictures, map, and others.

iii. Mechanical Learning Aids: This includes an audio learning machine, tape recorder, radio, projector, film strips.

iv. Audio-visual Learning Aids: This type of learning aids which includes video, cassettes, film, television, photographic slides

v. Visual Material Learning Aids: This includes charts, organization charts, flow charts etc.

vi. Mathematical Games: games puzzles in mathematics to stimulate interest and encourage self-thinking and learning in the students

vii. Mathematical Laboratory

8.6 Criteria for Selecting Learning Aids

The following criteria are suggested in the selection of learning aids for mathematics teaching-learning:

i. Relevance: Learning aid must be relevant to the subject matter and serve as a means of assisting the student learning on the particular subject matter presented. It should be considered from the point of view of actual helpfulness to the learner.

ii. Learner's ability: Before choosing a learning aid for a planned subject matter, the intellectual ability of the class is taken into consideration, so that it is not too advance or too simple. Furthermore, variety in types of aid is necessary because of the individual differences among students.

iii. Availability: Materials should be available and the aids should be numerous enough to permit selection by both teacher and learner; their helpfulness or lack of it should be looked into.

iv. Simplicity: Learning aid should not be too numerous. The teacher should not attempt to use many aids so as not to confuse the students by their multiplicity and rapid changes. An adequate and varied an indiscriminate and free use of them.

v. Cost: Learning aid should be economical. They should not be too expensive and too difficult to prepare, so that much time and energy would not be wasted on the part of the teacher.

vi. Accuracy: It should not be mere attraction. They should not attract attention to themselves but should increase interest in any comprehension of the subject matter to be learned.

vii. Usable: Learning aid should be readily usable. They should not take too much time and to keep them ready for use. They should also be able to last for considerable length of time without losing any of effectiveness.

viii. Objectivity: It should be adapted to the subject matter and to the goals to be secured through the mastery of subject matter.

ix. Size: Before choosing a learning aid for a planned lesson, you must considerable the size of the aid and also class size.

x. Teachers 'ability: If the learning aid is relevant to the subject matter but the teacher may not be able to operate or explain its application. In this situation, the student will not acquire the right knowledge from the lesson. Teacher must consider his/her ability before choosing any aid.

xi. Complexity: Some learning aids are very complex to explain even when the teacher known it. This type of aid is not easy to be comprehended by learners, so teacher should endeavor to avoid these complex learning aids.

xii. Durability: Some learning aid can be used in one or more attempts and require replacement due the materials used to make it, and storage facility. Such are not usually expensive and care must be taken to choose a more permanent and durable be stored and

8.7 Needs of Learning Aids

Mathematics teachers use a variety of aids to make the process of teaching and learning simple, interesting, no abstractness and effective which makes it easier to teach and learn even the most difficult concepts in the subject. The following are some of benefits of using learning aids:

i. Learning aid makes sense and saves time as students learn very quickly by watching and doing rather than reading. Therefore, teachers use aids.

ii. There are many students with a tendency to forget easily. Such students can get benefits with learning aids.

iii. Using learning aids will assist in understanding the concept easily and also grasp it completely.

iv. Learning aids enhance the conceptual thinking of students

v. Using learning aids createsan environment for motivation and interest in the learning process and does not learning concepts boring.

vi. With using of learning aids, the learners can learn with accuracy and even faster.

vii. It is proved that learning with visual representation stays in memory for a longer time than with textual representation. It impacts better with direct experience.

8.8 Practical Lesson

Below provide locally made learning aids and their objective in a lesson.

Topic 1: parallelograms on the same base and within the same parallel are equal in area.

Material: cardboard and drawing board procedure:

Draw on a piece of paper two parallelograms on the same base and within the same parallel. Pin the diagram on a drawing board having the same area with the parallelogram ABCD separately

Result: Turn the parallelogram ABCD

Over and place it on the parallelogram ABEF, you will discover that ABCD and ABEF are equal in area.

Topic 2: The Exterior angle of a triangle is equal to the sum of the interior opposite angles.

Material: Card Board and drawing board

Procedure:

Draw a triangle on a piece of paper and produce one of the lengths to obtain an exterior angle. Pin diagram on a drawing board. Mark out the interior opposite angles cut out two angles using a separate paper. Arrange the two angles on the exterior angles and the angles are found to be equal to the exterior angle. See diagram below:

Result: The sum of angles A and B when arrange on angles BCD are found to be equal.

Topic 3: To calculate areas of (i) square (ii) rectangle (iii) trapezium (iv) parallelogram (v) regular polygon

Procedure: Rule the plywood like graph paper with square of unit length. Nail each point where two lines meet. What you obtain is known as a Geo board and can be applied in a number of ways.

A GEO BOARD

To calculate the area of a square of length 4 unit string to carve out the area and count the number of square holes to give the area in square units.

For the area of trapezium, count the number of unit squares. Combine two half square to give the required area. In the trapezium given here the area = 8sq. units

ii. For the of parallelogram, count the number og square and combine two half units and add up. In this parallelogram the area = 6square units. Similar method is used for area of polygon etc.

Topic4: Longitude and Latitude

Objective: To prepare a learning aid for teaching latitude and longitude

Material: Palm frond for weaving basket or iron rod

Procedure: Make two small circle of the same radius from the material by tying. Make a third and bigger circle to represent the equator. Make another six circle of the same radius and bigger

than any of the first three circles Erect a rod longer in one of the six circles, ensuring that the length of the rod is longer than the diameter. Tie the remaining five circles in a vertical form at uniform intervals there represent the longitudes, fix the equator and the other circles as latitudes as shown below and tie the joints with string.

Note: when this material is not available, you can use iron or flex wire bent into this shape.

Topic5: Rectangular base pyramid.

Objective: To prepare a model for rectangle base pyramid

Procedure: Draw the figure on the cardboard as shown in the figure below and fold it up to close, by using gum after cutting through the edges.

Below see, before and after:

a. Before folding.

b. After folding (Rectangular pyramid)

Topic 6: Place value.

Objective: To prepare model of abacus for counting leading to place value

Procedure: Construct a rectangle frame of size 30cm by 20cm from the plank.

The thickness of the wood may be two centimeters. Choose two opposite frame and punch them in three places of equal intervals Arrange the beads on string on the frame as shown below:

Abacus

Topic7: plane shapes (circle, triangle, and square)

Objective: to identify various sorts of shapes I Their qualities and (ii) the representation of various planar shapes.

The material is either cardboard or plywood.

On cardboard, these basic forms are traced or created and then cut out.

Meaning and Scopes of planar forms

Generally speaking, plane forms are flat figures that are also referred to as two-dimensional (2-D) shapes. This category includes materials such as table tops, chalkboards, cardboard, ceiling board, circular objects, etc.

This lesson focuses on the geometric forms circle, polygon, and quadrilateral.

Circle: the location of (or the route traced by) a moving point that keeps the same distance from a fixed point. The measured distance around this route is known as its circumference. Other qualities of a circle are clued.

Radius, diameter, arc, chord, sector, segment

Polygon: A polygon is any plane shape bounded by straight line segment and it must not cross one another so that a polygon has only interior angles. Examples of polygons:

Pentagon Hexagon

Quadrilateral- it is a plane shape enclosed by focus line segments. Generally quadrilateral is a four sided shape.

Examples of quadrilateral are the parallelogram, rhombus, kite, rectangle, square, and trapezium.

Students Activity

1. Define a circle.
2. Outline all the properties of a circle. Student Activity
3. List 10 local materials that can be used to prepare learning Aid
4. Name the material you can use locally to prepare the following learning aids

(i) Ruler

(ii) Elliptical shape

(iii) Parabola frustum of a core

(iv) Cylinder

(v) Triangle prism

(vi) A hexagon

(vii) Sphere

5. Describe fully how you will prepare a cuboid using local material.

Parallelogram: is a quadrilateral (four side figure) made up of both pairs of opposite side that are parallel.

§ Opposite side are equal

§ Opposite side are equal in length and parallel

§ The diagonal bisects each other.

§ Each diagonal bisects and formed into congruent triangles

§ Allied angle are supplementary i.e. $<A+<B= 180^o$

§ $<B+<C=180^o$ $<C+<D=180^o$

Trapezium-is a quadrilateral having a pair of equal opposite side parallel. If the knowledge of trapezia is useful in dealing with cross-section of cones, pyramids prisms and other 3-D shapes

Special kinds of parallelogram.

There are special kinds of parallelogram. These are rhombus, rectangle and square.

A square is a parallelogram, which has four equal sides. A square has the following angle:

(i) The angles are all 90^o

(ii) All the sides are equal

(iii) Diagonals are equal

(iv) Diagonals bisect each other at night angles

(v) Diagonals bisect the angles.

$<A=<B=<C=<D=90^o$

AD=DC=CB=AB

The **rectangle** is a parallelogram in which all four angles are right angles (90^o). The rectangle has the following properties

(i) The angles are (90^o)

(ii) Diagonals are equal in length

(iii) Opposite sides are equal

Rhombus is a parallelogram in which a pair of adjacent sides is equal. A rhombus has the following properties

(i) Sides are all equal

(ii) The diagonals bisect each other at right angles

(iii) The diagonals bisect the angles

Kite-it is quadrilateral with two pairs of equal adjacent sides. It should be mooted that in a1 kite the opposite sides are not equal and it is not a regular quadrilateral because neither the angles nor the sides are equal.

Generally, a kite has the following properties

i. It has 2 pairs of equal adjacent sides AB=AD and CB=CD

ii. The diagonal BD and AC meet each other at right <AOB=90^o

iii. The correct diagonal cuts it into two isosceles Ds

Topics 8: SOLID SHAPES

Objectives: kinds and properties of solid shapes

ii. Modeling of solid shapes.

Introduction

In your secondary school days, you might have been learnt different kinds of shapes. Today you will look into solid shapes of 3-dimensional figures. Generally, solid are hard object figures. They have the following properties.

(i) They conform to various shapes or possess number of surfaces

(ii) They are three dimensional

(iii) The cannot be squeezed

(iv) They are not fluid

Examples of solid objects include, stone, cube, cone, cuboids, cylinder, blocks, tine of different shapes etc. Properties of solid shapes

Cuboids- it is a 3-dimenional solid with six rectangular sides. They are shapes like cartons of chalk; wooden boxes, cement blocks, match boxes, textbooks, empty packet of sugar. A cuboid has 6 faces, 8 vertices and 12 edges.

Cube-it resembles cuboid in all respects that cuboid has longer dimensions than cube. They are solid objects having 6 equal faces, 12 edges and 8 vertices. Examples are: cube of sugar, Maggi cube, dice etc.

Cylinder- is round object having round faces (curricular faces) and flat tops. Examples include: tin of milk, a lead pencil, a water pipe, a drum, a cylindrical wood etc.

Spheres- there are shapes that look like the shapes of egg, balloons, tomatoes and water pots.

Cones-it is circular based object having only one vertex, one circular edge and two faces. It can be referred to as a circular pyramid. Examples include the sharpened end of a drawing pencil, a kerosene funnel, and a top of salt measure.

Tetrahedron-it is a solid object having four faces, six edges

ii. Modeling of Solid Shapes

Material: Making these models you need calendar, cellotapes, gum, pair of scissors cardboard, wood glue, nails etc.

Activity: You ask the students to bring along all the requirements for laboratory practical. You now cut out the shapes using scissors or blade and a cardboard. Each time you are cutting a shape, let the student observed the process from beginning to an end. Give every individual student opportunity to do the same thing processes and give necessary assistance and motivation for self-learning. As a teacher, try to demonstrate different shapes on how to form geometrical shapes.

8.9 Summary

In this chapter, you have learned:

§ Improvisation is an act of using local materials to construct instructional aids in teaching mathematics. Local materials are materials obtained at home or school environment, such as empty bottle, mathematical set instruments, cardboard papers, ply wood, nail, ruler, iron rod, knife, saw, etc.

§ Learning aids is any form of materials that can aid or speed up learning of mathematics with or without any assistance of second person (mathematics teacher)

§ Instructional materials are those resource materials used by mathematics teachers in the classroom

§ Classification of learning aids are traditional, visual, mechanical, audio-visual, mathematics games, mathematical laboratory

§ Criteria for selection of learning aid relevance, learner ability, usage, availability, accuracy, durability, class size, teacher ability and complexity

§ Practical lesson was discussed by defining the plane shapes, solid shapes, their properties and modeling.

Student Activity

1. Define a polygon
   a. Quadrilateral
   b. A parallelogram
   c. A trapezium
   d. Square
   e. A rectangle
   f. A kite
   g. A rhombus
2. Outline the properties of
   a. Rhombus
   b. A square
   c. A kite
   d. Polygon
   e. Quadrilateral
3. Draw the following solid shapes.
   a. Cuboids
   b. Cone
   c. Cylinder
   d. Tetrahedron

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CHAPTER NINE

QUESTION TECHNIQUE

9.1 Objectives

At the end of this chapter, you should be able to:

i. define the word question
ii. explain the types of question
iii. list and explain the characteristics of good question
iv. state the principles that teacher observe when asking oral questions
v. discuss the functions of mathematics question
vi. define exercise in mathematics
vii. discuss the functions of mathematics exercise
viii. enumerate and explain the characteristics of mathematics exercise

9.2 Introduction

The termed questioning method or Socrates method is named after the Greek Philosopher Socrates (470-399 BC) who had reputation of asking questions to make his points. From childhood, man asks question for various reasons among which is to find out or learn new experiences. In mathematics classroom, questions are very vital tools that the teachers and students utilize to teach or to learn.

In this chapter, the characteristics of good question are discussed and also the types of question. Mathematics exercises in mathematics classroom are examined in detail. Emphasis will be given to its functions, purposes and procedures of asking questions and finally the characteristics of exercises.

9.3 What is Question?

Question in a simple term is a linguistic expression used to make a request for information or request made using such an expression. The information requested is provided in the form of an answer. Oral questioning is another very powerful technique in the classroom for the mathematics teacher interacts with the students. It involves the students in the session through thinking and provides you feedback on the level of learning. This question session may be at introduction stage, presentation stage or at conclusion stage. The questions asked by the teacher at any of these stages are not a basic method in the lesson but a means of providing students' adequate facts and making the facts clearer and understandable. In another direction, questioning may be employed as method of instruction. If the mathematics teacher is very skillful handling of questions, teacher can make an effective and efficient lesson.The technique should not be used for long duration to avoid discouragement from part of the students.

9.4 Types of Questions

Five sorts of questions have been found and divided into two categories: lower questions and higher questions. According to some experts, the lower questions are factual inquiries, whereas the upper ones are thought-provoking questions. These five (5) varieties of inquiries:

1. Presentation or Teaching Questions: These are utilized during the lesson's introductory phase. They serve to make the instructor and students aware of the topic content.
2. Pause Questions: Are these types of questions employed to give a helpful pause throughout a class and ascertain that pupil are paying close attention?
3. Guide Questions: These questions are designed to draw the student's attention to particular aspects of the course. Such questions facilitate keen observation, deft correlation, and precision.
4. The purpose of summary questions is to review and assess the material delivered.
5. Drill Questions: These are questions asked at the beginning, throughout, or at the conclusion of the course, and they should be brief, no more than 5 minutes. Such questions require the learner to be vigilant, for instance mental arithmetic sums.

9.5 Qualities of Effective Questions

At all three phases of class education, the primary reason for asking a question is to obtain a response. However, this may only be accomplished if the instructor considers certain actions throughout the question-asking phase. These include:

i. Clarity: The query should be concise and direct. That is, pupils should be able to comprehend the language used in the questions, which should be simple and familiar.

ii. Definiteness: The inquiry must restrict the scope of generalizations. As a general rule, questions demanding YES or NO should be avoided as much as possible, as well as elliptical questions, which encourage students to speculate. The question must provoke thought and demonstrate application of acquired information.

iii. Interest: Questions must be engaging and should elicit zeal and a quest for knowledge.

iv. Fairness: Questions should be given fairly so that students who are modest, timid, or unmotivated can be encouraged to think critically.

v. Capability: Questions must be at the students' level. It must be something that pupils can comprehend and undertake.

9.6 Principles of Question Technique

The following are concepts of question approach for interrogation. These include:

i. Questions must be properly crafted so that students answer with their knowledge and not just yes or no.

ii. Begin with easy questions, then progress to complicated ones.

iii. Distribute the questions so that every student may reply.

iv. Commend a proper response and investigate the cause of an inaccurate response. Always accept the student's response to encourage engagement in the future.

v. Avoid discouraging pupils by not ridiculing incorrect answers or the remaining students.

vi. Give the learner time to reflect on the questions posed.

vii. Explain unknown mathematical terminology, phrases, or words to the pupils.

9.7 Uses of Mathematical Question

As tedious as mathematics may appear, her research has implications for education and our everyday lives as follows:

§ Mathematics problems help us develop analytical thinking.

§ Analytical thinking fosters the ability to research and discover the truth about the surrounding world.

§ Mathematical questions cultivate the ability to reason. Because to locate the solutions, you must consider a comprehensive, cohesive procedure. It may be claimed that mathematics is essential to a child's education since it teaches them to think.

§ Mathematical questions enable us to articulate our thoughts and ideas with clarity, coherence, and precision, allowing us to understand how things function.

§ Mathematical questions boost intelligence. Mathematics is applicable to various disciplines, such as technology, and is pervasive in everyday life.

§ Mathematical questions stimulate the intellect and help us to think more deeply and critically when confronted with complicated challenges.

9.8 What is the definition of Mathematical Exercise?

A mathematics exercise consists of problems whose answers need just regular methods. A learner who has mastered the required skills will be able to follow a clear plan and sequentially apply the techniques in simple stages to arrive at the proper result.

9.9 Qualities of an Effective Mathematics Exercise

Examples of effective mathematical activities are:

§ Foundational Concepts: What are the fundamental mathematical notions or concepts that students must understand?

§ Mathematical Abilities: What abilities must students possess to demonstrate proficiency?

§ Important Questions: What are the essential questions that connect the fundamental ideas with the necessary skills? Students' contributions to the problem will provide answers to the crucial questions.

§ Statement Prompt: Provides students with a phrase or paragraph describing the problem and the solution.

9.10 The Purposes of Mathematical Exercise

i. A successful mathematical exercise increases mathematics learning and becomes a routine component of continuous classroom activity, as opposed to an interruption. Assessment is not only the culmination of a learning cycle. Rather, it is an integral component of instruction that promotes and facilitates further study. Listening to pupils, observing them, and interpreting what they say and do are included. Observing pupils' work, especially with very young children, can reveal elements of thinking that are not touched by written or oral activities.

ii. A good mathematical exercise also aids in lesson planning and instructional decision-making, allowing teachers to identify multiple assessment opportunities. Questions such as the ones below become a standard part of the educator's planning: "What questions am I going to ask?" What will I be observing? "Which activities are most likely to inform me about students' learning?"

iii. A good mathematical exercise promotes mathematics learning by incorporating tasks that are congruent with, and occasionally identical to, those used in teaching. For instance, if students are learning through articulating their mathematical concepts in writing, their mathematical knowledge is tested in part by requiring them to write about their mathematical ideas. If students' study in groups, they may also be evaluated in groups. If graphing calculators are utilized in the classroom, they must be available for evaluation.

iv. A strong mathematics exercise, in addition to projects and other out-of-class work, provides a rich source of evaluation data for drawing conclusions about students' development. Numerous results of classroom engagement, including spoken remarks, written papers, drawings, and computer-generated models, are indications of mathematical learning. Together with information from more formal assessment activities, students and instructors utilize this data to identify the next stages in learning. Activities ranging from a rough draft that demonstrates the utilization of feedback and constructive criticism as a means of improvement to a polished final result provide evidence of mathematical learning.

Student Activity

i. What is Exercise in mathematics

ii. List the characteristics of good exercise in mathematics

iii. What are functions of questioning?

iv. List all kinds of questions you know and explain two

v. State four characteristics of good questions

vi. What are principles a mathematics teacher should follow when asking question?

9.11 Summary

Questioning is process in which the teacher makes a statement and asks higher order to lead the learner to see the limitations in such learner. However, five types of questions areidentified; presentation or teaching, pause, guide, summary and drill questions. Equally, the question assists the teacher to know the previous task learnt. A good question is expected to be clear, definite, interest and fair distributed.

A mathematical exercise is questioning whose solution involves only routine procedures. Good exercise is expected to involve core concept, essential, mathematical skills and statement prompts. A good mathematical exercise enhances better learning and assists in planning lessons and also making instructional decisions.

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CHAPTER TEN

PROBLEM SOLVING

10.1 Objectives

At the end of this chapter, you should be able to:

i. define and explain the term mathematical problem
ii. list the characteristics of mathematical problem
iii. explain the classification of mathematical problem
iv. define problem solving
v. enumerate attributes of problem solver
vi. discuss briefly problem-solving models

10.2 Introduction

In this chapter, the meaning of mathematical problem, characteristics of mathematical problem and problem solving are emphasized. Also, in this chapter classification of mathematical problem, qualities of good problem solver and importance of teaching problem solving were clearly explained.

10.3 What is Mathematical Problem?

Some mathematics students and individuals regard every mathematical question and textbook practice as a problem. However, this definition of a problem is not expected by many mathematics instructors since it does not take into account the nature of the individual confronting a particular difficulty and the situation or moment in which they find themselves.

Nonetheless, some mathematics instructors defined a mathematical problem as a circumstance for which there is no procedure that guarantees a solution. The relevant information of the individual must be recombined in a novel manner to address the challenge. This description is consistent with Godwin (2006), who defines a mathematical issue as "a circumstance in which an individual or group is required to undertake a job for which there is no easily available algorithm that totally dictates the solution approach."

This definition suggests that an issue is more than just a query or a circumstance; it must also be puzzling. Consider the query: what is the perimeter of a square that is 5cm long and 5cm wide? This question should not provide any difficulty for SSII students. A question or predicament is a sort of phrase that demands or seems to require a response. Consequently, a subject or circumstance can only be deemed confusing and thus an issue in respect to a person and a period. In the past, this question presented a challenge for every student in that class, but it no longer does.

10.4 Mathematical Problem Elements

The elements or qualities of a mathematical problem are:

§ *Acceptance:* the individual or student recognizes it as an issue.

§ *Blockage:* that the individual or student cannot quickly solve the problem with his or her first try.

The individual's drive and desire to address the problem by investigating other solutions.

As obvious as it may seem, a mathematical problem is a novel situation involving perplexing simple and compound questions about number, space, or relationship that can be answered by the essential utilization of necessary information given directly or indirectly in an analytical procedure that is purely interpretive, logically argumentative, computational, and otherwise dependent on mathematics. Numerous mathematics instructors have praised this concept, which distinguishes exercises from problems in a variety of ways. That exercises are utilized in mathematics class education to offer practice in previously taught learning abilities or as applications of comprehension. In contrast to exercises, problems demand the student or individual to use synthesis or insight. To solve a problem, a student must rely on previously acquired information, abilities, and understandings, but he synthesizes prior learning while applying them to a new context.

10.5 Classification of Mathematical Problems

Even in antiquity, it was believed that certain pupils possessed great intelligence, medium intelligence, and low intelligence. Previously, however, the term intelligence had little psychological value. Gall, Lavator, Benjamin, and others have identified six (6) degrees of intellectual load that a normal problem places on a human problem-solver. The challenges may consequently be divided into six categories requiring solutions: Knowledge is the ability to recall specific information, terminology, steps and sequence, categorization, and criteria. Understanding and interpreting information is comprehension. Application: the use of a principle or body of information to solve issues Analysis is the process of breaking down information into its component elements in order to identify a link or principle. Synthesis is the process of combining components and pieces to make a whole. Evaluation: a determination based on internal and external information as well as requirements for precision.

Polya saw assessment as looking back to assess the relevance of the problem, the adequacy of assumed facts, relations, and methods, the adequacy of all logical, mathematical, or algebraic processes, as well as the accuracy and reasonableness of the produced answer.

According to Akpan (1989) in Godwin (2006), mathematics issues are categorized as mixture problems, distance-rate and time problems, real-world questions, number theory difficulties, proof open-search challenges, and so forth.

This domain was initially published by Bloom et al. and focuses on problem-solving (1956).

10.6 What is the Definition of Problem-Solving?

According to Polya, problem-solving is finding a way around an impediment when none is immediately apparent. He also argued that to know mathematics is to solve difficulties. Problem-solving is the methodical application of previously learned information and abilities to the demands of a new circumstance. This method combines meticulous investigation of a particular circumstance with the synthesis of previously learned information in order to discover a solution to an issue. Critical thinking is a crucial skill for issue resolution.

Educators of mathematics stressed that knowledge is a collection of problem-solving techniques. The mathematics educators' definitions were congruent with the NCTM's definition. Particularly, each and every mathematics instructor emphasized the importance of non-repetitive issues. As long as the learner is unfamiliar with the solution route, everything the student undertakes to arrive at a solution qualifies as issue solving. The majority of responses ended with this demand. In addition to the non-routine aspect of the issue, some mathematics instructors specified other requirements. These extras increase adaptability and independence of thought. The solution path must be within reach of the learner, there must be a "desire or need to find a solution," logic and knowledge must be employed in the process of solving, and there must be a "want or need to find a solution." In addition to the ability to reach a solution, motivation, a process of reasoning, and the use of existing information, the additional requirements listed by the mathematics educators included the possibility of reaching a solution, motivation, a process of reasoning, and the use of existing information.

In conclusion, from the perspective of a mathematics educator, problem solving is the process by which a student resolves a situation for which the technique of approach is unknown or not clear (non-routine) to the learner.

10.7 Who is an Effective Problem Solver?

A proficient problem-solver possesses the following qualities:

- They do not need to always be correct.
- They surpass their own predispositions.
- They search for opportunities inside the issue.
- They can distinguish between complicated and simple thought.
- They have a good understanding of the situation.

§ They consider their alternatives.

§ They possess realistic anticipations.

10.8 Importance of Problem-Solving Instruction

According to mathematics educators, the major goal of mathematical problem-solving teaching is not to provide pupils with a collection of skills and procedures, but rather to foster independent thought. The value of training in skills and processes should be determined by the extent to which the skills and processes foster flexible, autonomous thought. Aliyu provided the following as examples of the significance of teaching problem solving: Integration of prior knowledge: Through problem solving, students may apply and integrate previously taught concepts, generalizations, and abilities. The ability to solve problems is one of the most valuable benefits someone may receive from the study of mathematics. Students are able to solve common problems successfully and critically reason. Pupils' Development: problem solving enhances students' mathematical reasoning and communication skills. It offers the intellectual environment for mathematical growth and transforms the learner into a junior researcher in mathematics. Problem-solving provides pupils with the opportunity to discover mathematical formulas and techniques. It is considered to be the most fundamental mathematical activity. Problem-solving is the foundation for other mathematical processes such as generalization, abstraction, theory construction, and idea development. Class interaction: problem solving increases relationships between students and between students and the teacher. Consequently, performance and response in mathematics improved. Creativity of facts: problem-solving is a prerequisite for innovation in mathematics and encourages pupils to become more analytical in their decision-making abilities. It is an excellent preparation for advanced mathematical studies.

There are key parts of mathematical material that enhance or facilitate problem-solving in the discipline. Facts, concepts, definitions, symbols, axioms, principles, operations, algorithms, and theorems, according to Godwin (2006), are fundamental aspects that facilitate problem resolution.

It provides a solid natural foundation for verifying a broader problem-solving scenario. 2+6=8

This symbolizes the items or elements under consideration in the problem, such as a triangle with a right angle.

This gives appropriate meaning for the problem's ideas and facilitates problem understanding. The following are examples of Integer properties: Vacant property Community Property Inverse Quality Other cumulative properties

Symbols communicate distinct meanings and aid in issue interpretation; for instance, f(x)dx will evoke an integration reaction and not a differentiation response.

Axioms are the fundamental and defining characteristics or laws of a mathematical system.

Principles - There are general rules that aid in problem-solving by determining some essential steps and work patterns in advance and providing a standard approach. For example, the mathematical induction principle. If p (1) is true and p (k+1) is true, then whenever p (k) is true, p (n), a positive integer or natural number, is also true.

Operations- There is specific basic functions for changing one, two or more items of a set at once to a single element. It gives issue solution with legitimate procedural strategies for reducing complicated expressions to their basic versions. E.g. (24×3)/8=9 here 24, 3 and 8 have become 14.

Algorithm- There is steps to be followed to address a given problem.

Theorems- It constitutes the ultimate objective of all pure mathematical endeavor. In regular or non-routine application problem solving, they offer circumstances to hunt for or to develop, so that a beneficial truth may assumed or employed more directly.

10.9 The use of heuristics in problem solving

It is any strategy to problem-solving, learning, or discovery that involves a method that is not guaranteed to be ideal or faultless but is enough for achieving immediate objectives. Heuristic objectives are not formal problem-solving models, but they can be utilized as an approach to problem-solving when flawless or optimum solutions are not expected. Heuristics are often mental short-cuts that facilitate the problem-solving process.

It is a mental shortcut that enables individuals to solve issues and make decisions fast and effectively. It helps us uncover the most efficient and effective solutions to challenges. Mathematics encompasses much more than the study of numbers, spaces, and patterns.

While an algorithm is a series of steps designed to solve a certain issue. Heuristics play a significant part in the problem-solving process. Heuristics exist because, most of the time, they facilitate the discovery of a straightforward solution to difficult issues. In the case of insight difficulties, heuristics are not a poor problem-solving technique. Indeed, an innovative and creative approach is the only way to reach a solution.

10.10 Methodologies and heuristics for problem resolution

There are two (2) problem-solving methods: the pure discovery method and the guided discovery method. The pure discovery method relies on the student's prior knowledge to approach the problem without the teacher's guidance. This strategy has benefits and drawbacks. The majority of the time, the student wastes a great deal of time, but he may discover more concepts than the teacher anticipated. Oftentimes, the concept discovered is not the one anticipated, and the concept discovered may be incorrect. It is a very

challenging approach and an incorrectly discovered concept. Therefore, this practice is discouraged. The following are examples of pure discovery approaches falling under this category

a. Intelligent Guessing Discovery Approach

The following are suggested simple steps to approach a problem, intelligent by the students without involving the teacher:

Step 1: Search for entry cues at every level

Step 2: Apply simple rules of arithmetic such as if A or B is digit, then

i. A+A is always even

ii. A+B cannot be more than 18

Step 3: Do not be afraid to guess an answer

Step 4: Test your result. This process is very useful in the solution of cry arithmetic problem such as:

Example: If ZOO + Z =BEE what number do these letters represent

Working

Apply the steps suggested above,

For a cue, notice that in the ten's digit, no number is added to 0 to get another letter E, this means that there must be a "carryover",from the unit column.

Again, since the letters O+Z can never be more than 18 the number carried must be 1.

Then 1+0=E the only digit to be added to 1 to have a carryover is 9.

0=9 if 0=9 then E=0(zero) since 1+9=10

Then Z, must be 1

The solution is 199+1=200

Z=1, 0=9, E=0, B=2.

ii) Solve the following problems using alpha metric(cryptarithmetic) approach in which each letter represents a unique digit.

```
  HAVE
+ SOME
------
HONEY
```

You can adopt the steps provided to verify this solution.

Sol: 1486+9076=10562 thus, H=1, E=6, N=5, M=7, S=9, O=0

iii)

```
  SPEND
- MORE
------
MONEY
```

Sol: S=6, P=4, E=7, N=9, D=0, M=5, O=8, R=1, Y=3 which gives: 64790-5817=58973

Iv)

```
  SEND
+ MORE
------
MONEY
```

Sol: S=9, E=5, N=6, D=7, M=1, O=0, Y=2, R=8, which give: 9567+1085=10652

b. Solution by Intelligent Elimination

Consider the following problem WAEC, June, 1999 question objective

Example 2:

It is observed that

$1+3 = 2^2$

$1+3+5 = 3^2$

$1+3+5+7 = 4^2$

$1+3+5+7+9 = 5^2$

If $1+3+5+7+9+11+13+15 = p^2$ find p

(A) 6 (B) 7 (C) 8 (D) 9

An intelligent solution by elimination goes like this. Sum is always a perfect square of all the options only C is a perfect square of number of terms, 8. Answer is C.

Group Two

Guided discovery Approach is a process of instruction in which the student discovers the concept by himself under the special directives given by the teacher. The directive may be in form of:

a) Socratic question
b) Teacher initiated activities
c) Discovery chart
d) Inductive reasoning
e) Deductive reasoning

a. Socratic Questioning Approach

Example 3: how would you discover that for any number, $n^o=1$

Working: consider the following dialogue between the teacher (T) and the student (S_1)

T What is the meaning of 3^0?

S_1 It is zero, sir

T Do you agree with the answer, S_2?

S_2 I don't know, sir

T What is the value of a number divided by it?

S_2 It gives 1

T Good, then 3^2 Divides $3^{2:}$ gives what?

S_2 1, sir

T Very Good, what do you think 8^0 will be?

S_2 It will be 1 also

T Very Good, what about any number, n^0

S_2 1 sir

T Good, what is your conclusion about the value of any number raised to power?

S Any number raised to power zero is 1

b) Inductive Reasoning:

Is a process examining particular cases for a pattern and coming to generalization from that pattern.

Example 4:

consider the following and answer the question that follows: $14^2-13^2 = 14+13$

$27^2-26^2 =$ $27+26$

$39^2-38^2 =$ $39+38$

$199^2-198^2 =$ _ _ _ _

Do you agree with this, check this it is true?

What pattern do you discover? Can you generalize with any number n, the formula?

If you are able to come to generalization from the pattern, then this is an example of inductive reasoning. The generalization is that for any number n: $n^2-(n-1)^2= n+(n-1)$. Inductive reasoning is very important in prediction, in planning, in budgeting etc.

Student Activity

1. Given that p=8, find values for these letters so that AT+PT=FEE (All letters are one-Digit numbers)
2. Examine the sequence of number; 1,1,2,3,5,8,x,y,z. find the values of x,y,z.
3. Apply Socratic questioning approach to set a student discover that $a^{-1}= 1/a$
4. It is observed that

$2+4 = 2^2+2$

$2+4+6 =3^3+3$

$2+4+6+8 = 4^4+4$

If $2+4+6+8+10+12+14+16= p^2+p$, find p.

10.11 Deductive Reasoning Approach

It is a process of giving deductive logical argument in which all the reasons call premises are accepted as true and the conclusions necessarily follow from the reasons. This leads to mathematical proof in geometry or other branches of mathematics. In geometry proof, the format is made up of four parts: the givens, required to prove, the construction (If any) and the proof. All these techniques are contained in the senior

secondary mathematics and further mathematics curricula. Each of these parts must be learnt separately in one full lesson or more.

a) Givens: Given include explicit and implicit givens. Explicit givens are data stated and easily seen contained in the problem implicit givens are made up of pieces of knowledge, information, theorems definitions and previous knowledge implied in the problem, not stated but are necessary to unable you solve the problem.

b) Required to Prove: In identifying the givens, you also required to identify the unknown or what you are required to prove in the problem.

c) Construction: A combination of explicit and implicit givens usually leads to the required construction and statements, which are premises to the conclusion which we call.

d) The proof.

10.12 Definition of Mathematical Proof

A mathematical proof is a valid argument using axioms or already proved true statements to establish the truth of a proposition. A proof is therefore valid argument in which the conclusion is necessary derived from a set of true statements known as **Premises.**

An argument is a mathematical proof if and only if it satisfies the following two conditions.

i. All the premises are true, and

ii. The argument is valid and leads to a valid conclusion.

10.13 Kinds of mathematical proof

i. Deductive proof: this is the usual formal proof used in geometric proof. e.g. prove that the sum of angles of a triangle is 180

ii. Conditional proof: the argument is of the form" if p…then q". E.g. if a line is perpendicular to a radius of circle at it point of extremity, and then the line is a tangent.

iii. Bi-conditional proof: this type of proof contains two conditional statements. Take example of this statement "quadrilateral is a square if and only if all the four sides are equal and one of the interior angles is a right angle"

iv. Proof by counter example: the general statement is usually used to prove or disapprove. If one example can be found which obeys all the premises but makes the general statement wrong, then that example is a counter example, and it proves the general statement wrong.

Example: the general statement "any 4 points in a plane form quadrilateral when joined in order is proved wrong if any 3 of the points be in a straight line.

10.14 Problem solution in mathematics

Some discussions would be made to guide you in finding solutions to mathematical problems, especially in the areas of proofs in geometry, variations, proportion.

Example 1: Prove that the sum of angles of triangle is 780°. Write down all the explicit and implicit given, required to prove and construction in the problem.

Working

Explicit given include:

i. Any triangle
ii. The three sides of triangle
iii. The sum of the three angles
iv. The angle 180°

Implicit given include:

i. A specific triangle ABC representing any triangle
ii. The sum of the angles ABC+ABC+BCA
iii. The angle on a straight line is 180°
iv. When lines are parallel (a) Corresponding angles are equal

(b)Alternative angles are equal

The unknown is the required to prove that: BAC+ABC+BCA= 180°.

Construction: the explicit and implicit given suggested drawing any triangle. In this case, we have chosen to draw triangle ABC. You are free to draw another triangle PQR or XYZ. But once you have drawn a specifically named triangle, restrict your references to that triangle.

Based on given, we draw straight line at by producingBC to D to set the straight line.

– – – – – – – – – –

Proof: Now after considering the explicit and implicit given, the proof becomes very clear.

Variations

Direct variation: A relationship between any two quantities in such a way that is constant and called a direct variation OR we can also write where is a constant. A short form of writing varies directly as x is " "

Characteristics of Direct Variation

i. OR where is a constant.
ii. If is increased, y is also increased in the same ratio.
iii. If is decreased, y is also decreased in the same ratio.
iv. When the graph of direct variation is drawn, it is a straight line.

Working Examples

Consider the relationship between two quantities as in the discovery chart

2	3	5	4	7	6	1	10	11	m	X
6	9	15	12	-	-	-	-	-	-	-

a) Fill the blank spaces and answer the following questions

i. Discover the relationship between Ans: Y=3times x

ii. Increased what happens to y? Ans: Y is also increased

iii. is multiplied by 3, what happens to y? Ans: Y is also multiplied by 3

iv. Consider the ratio of for each pair of values. What do you notice? Ans: always.

v. What is the equation connecting Ans: Y=

b) Fill the following blank spaces

i. Y=----of x ii. X=----of y Ans: i. 3 ii.

c) Is it true that what is the constant Ans: constant =3

2. The cost (c) of feeding students per day is directly proportional to the number (n) of students. The cost of feeding 20 students a day is N800:00

a) What is the cost of feeding 36 students at the same rate?

b) What is the cost of feeding 80 students at the same rate?

Indirect or Inverse Variation

It is a relationship between x and y in which xy= constant, is called an inverse variation. The equation of an inverse variation is xy=k or where k is a constant.

Characteristics of an Inverse Variation

i. In an inverse variation or where k is a constant.

ii. If x is increased, y is decreased by the same ratio

iii. If x is decreased, y is increased by the same ratio

iv. If the graph of inverse variation is drawn, it forms a straight line bending backwards.

Working Examples

2. The number of days certain of quantity food lasts is inversely proportional to the number of students. The food lasts 2oo days for 100 students.

a) How long will the food last for 250 students.

b) How many students shall be retained to that the food lasts for 550 days?

Working: Let d stand for number of days and s stands for number of students now let d varies inversely as

S

when d=20, S=100,

Relationship between d and s is

a. When s=250, then d= food will last 8 days for 250 students.

b. When 0=50 then

Number of students will be 40.

Discovery Chart

It is a pattern chart in which the teacher has supplied some of the solutions and left some columns empty. The students are expected to follow the teacher's model to complete the missing columns and generalization.

Consider the discovery chart and answer the questions that follow.

12	6	2	1	3	16	48	X
4	8	24	48	-	-	-	-

i. If x is increased, what happens to y? Ans: y decreases

ii. If x is decreased, what happens to y? Ans: y increases

iii. If x is multiplied by 2, what changes occur in y? Ans: y is divided by 2

iv. Is it true that if x is divided by 2, y is multiplied by 2? Ans: yes

v. Calculate the product xy in each pair what do you notice

vi. Is it true that xy=constant? What is the value of the constant?

Conclusion: xy=constant =48 in each case.

Proportion

Consider this example

In Aliyu's family, there are 2 boys and 3 girls while in Ndagara's family there are 4 boys and 6 girls. Calculate the ratio of boys to girls in both families.

Working: Aliyu's family, boys is to girls as 2 is to 3 in short in Aliyu's family, boys:girls =2:3

We say in Aliyu's family the ratio of boys to girls is 2/3 (read 2 is to 3) similarly in Ndagara's family the ratio of boys to girls is 4:6 or in fraction 4/6

Now when written in a lowest term

4:6=2:3 0r we say that have the same proportion.

Definition: A proportion is a statement that two ratios are equal. In general is a proportion which can be written as a:b=c read "a" is to b as c is to d.

Terms of Proportion: In the proportion or a: b=c: d a is the 1^{st} term, b is the 2^{nd} term, c is the 3^{rd} term and d is the 4^{th} term. Thus, b and c are the mean term, a and d are the extreme terms. So the mean product is the product of the mean terms = b×c.

The extreme product is the product of the extreme terms =a×d.

In ordinary language, we discover that bc=ad, that is mean product= product of extremes. We call this rule cross multiplication. Thus, in the proportion we have that 2×6 = 4×3 i.e. product of means = product of extremes. Now, identify (a) the 1^{st}, 2^{nd}, 3^{rd} and 4^{th} terms (b) the mean product (c) the extreme product of the proportion .

Example 1: If find the value of m.

Working: product of mean = 14×m = 14m

Product of extremes = 18×21

Equating, we have 14m = 18×21

m=

Students Activity

1. Write down at least 2 characteristics of mathematical proof
2. Give one example of a conditional statement
3. Construct one example of bi-conditional statement
4. What type of statement is this? A rectangle is a square if and only if all the sides are equal
5. The distance, 5km. travelled by a motorist at a constant speed varies directly as the time, t hours. In time 5hours the motorist covered 400km, find

(a) The time he will cover at distance 960km

(b) The distance he covered in 8 hours. Give two reasons for saying that, this is a direct variation

6. Given that

(a) Identify the 1^{st}, 2^{nd}, 3^{rd} and the fourth terms

(b) Write down the mean product

(c) Write an equation connecting the mean product and the extreme product

7. If p:q:r = 2:3:4 evaluate(i) (ii)
8. Write down main differences between direct and indirect variation
9. Write down all the explicit and implicit given and use them to prove that the angles on the same segment of a circle are equal
10. Examine the sequence of numbers and find the missing numbers: 0, 3, 8, 15, 24, X, Y, 63, 80. Find the values of X and Y.
11. State two principles necessary for solving cryptarithmetic problem
12. Apply Socartic questioning approach to discover that = given that = and $\log_a M = x$, $\log_a N = y$ and $M=a^x$, $N=a^y$ use a named discovery approach to discover that $\log_a MN = \log_a M + \log_a N$.

10.15 Problem Solving Models

The following models in problem solving are as follows:

Polya's model

Scope's model

Krolik's model

Rubenstin's model

Polya's Model

Polya. G(1957) defined four phases for the solution of any problem.

Phase 1: Understanding the problem

Phase 2: Planning for the solution

Phase 3: Computation skills

Phase 4: Looking back.

*Understanding the problem:*Ability to identify the givens and the structure of the problem, and also ability to grasp the relationships among the given.

Planning for the solution: This involves translation the givens and their relations to form symbols, diagrams, equations or mixed models then using the givens and their relations to develop the algorithm necessary for the solution of the problem.

Computation skills: This involves a variety of ways of carrying out computations in the algorithm.

Competence in the skill is acquired through repetitive practice. Many students learn by solving similar and identical kinds of problems.

Looking back: This involves checking through the processes of understanding, planning and computation

Scope (1973): He proposed six steps for the solution of any problem.

Step 1: Rephrase

Step 2: Trial and error

Step 3: Search for pattern

Step 4: Insight

Step 5: Justify insight

Step 6: Generalization

Rephrase: Here the problem solver needs to restate the problem in a precise language, so that if there are ambiguities, there are eliminate.

*Trial and Error:*Applying trial and error to find way to the clue, by trying several approaches. That is if one's effort fails, let him proceeds to next.

*Search for Pattern:*The problem solver should begin to look for patterns which may link the present problem to others that he might have come across in the past.

Insight: Problem solver at particular times may not have solution to a problem at his finger tip. But at later time, the problem solver discovers the clue that will lead him to solve the problem at hand.

Justify insight: The problem solver must now utilize the approaches or logical reasons to back up his intentions.

Generalization: The problem solver now looks back on the completion of work to see if the results or the methods used to be generalized.

Robenstein (1975): defined four stages in problem solving. Which include: Preparation, Incubation, Inspiration and Verification?

*Preparation:*At this stage, the problem solver examines the elements of the problem and studies their relationships.

Incubation: At this stage, if the problem solvers search for a solution could no yields, no positive result. Let him put the problem aside and sleep on it for a while.

Inspiration: After sleeping on the problem for sometimes, the problem solver may suddenly get an idea or insight that will enable him to solve the problem at hand.

*Verification:*Here at this stage, the problem solvers look back to his solution to verify if it makes meaning.

Krulik and Rudnick (1980): the authors proposed five stages in problem solving. This includes: Read, Explore, Select a strategy, Solve and Review & Extend.

Read: The problem solvers read the problem, note keywords and know the unknown.

Explore: Here the problem solver draw the diagrams, making a chart a search for a pattern.

*Select a strategy:*At this stage, the problem solver experiment, think of simpler problem and make a guess to a solution.

*Solve:*The problem solver carryout the plan.

Review &Extend: At this stage, you verify your answer and look for interesting variation of the original problem.

10.16 Summary

§ Problem in mathematics and its characteristics are discussed.

§ Problem solving and qualities of good problem solver were outlined.

§ A mathematical proof is a valid argument applying axioms or already proved true statements to establish the truth of the proposition.

§ A direct variation is any relationship in the form A=KB where K is the constant. Inverse variation is any relationship in the form AB=K where K is the constant

§ Problem solving models were explained and

§ Phases involved in each of the model are well described

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CHAPTER ELEVEN

COMMON ERRORS IN MATHEMATICS

11.1 Objectives

At the end of this chapter, you should be able to:

i. identify and explain all types of commonerrors in mathematics
ii. give at least one example of each type of error
iii. know the Semantic and Syntactic Errors in some concepts

11.2 Introduction

Learning will be facilitated What is stated and proved is unambiguous and transparent. The art of communication, or the transfer of ideas from one mind to another, is a major determinant of the effectiveness of education. Gilbert Highet emphasized the need of clear communication in education, stating, "Let him (teacher) be good at communication, and even if he is a mediocre scholar, he may be an exceptional teacher." In another meaning, communication refers to the relationship between students and teachers. Studies often demonstrate a strong association between communication and good teaching-learning. Teachers of mathematics should avoid semantic and syntactic mistakes that negatively impact teaching-learning in mathematics; hence, each topic has its own technical language and terminology. As a field of study, mathematics appears to be primarily concerned with the precision of its solutions and has little to do with its medium of education. Inekwe (2003) discovered that if a child's language proficiency imposes a linguistic handicap, the expected consequence of what is taught would always fall short of the input. Clear grammatical structure during classroom instruction contributes to mathematical language comprehension. Examining the mathematical response scripts of students at all levels reveals at least eight types of mistakes. These include syntactic, semantic, pragmatic errors (Ahmad, 2010). Some other errors are errors in mechanistic understanding, cultural inhibition, premature approximation, misreading and computational errors. Some of these errors are in the nature of mathematics while other is due to undue emphasis or lack of emphasis placed by the teacher during the teaching process.

11.3 Types of Common Errors

Examination of students' answer scripts all levels mathematics reveals at least eight types of errors. These include syntactic, semantic, pragmatic errors (Ahmad, 2010). Some other errors are errors in mechanistic understanding, cultural inhibition, premature approximation, misreading and computational errors. Some of these errors are in the

nature of mathematics while other is due to undue emphasis or lack of emphasis placed by the teacher during the teaching process.

11.3.1 Syntactic Error

It is an error in the relationship between a symbol and other symbols in mathematical expression. For example, consider these expressions a. $p = 3q-5$ b. $p = p^3-5$ the two expressions contain the symbols p, q, =, -, 5, 3, but have different meanings in expression. 3 and q in first expression means 3 multiply by q while in second expression the relation is that of exponential or power.

11.3.2 Semantic Error

Semantic error is an error as a result of not taking into consideration what a symbol stands for or represent, in a mathematical expression. What a symbol stands for in an expression is known as Referent. Example, given that 7/x find the value' x" the variable cannot be zero. As student who recorded that $7\div x=7$ or 0 that student committed the error of semantic because division by zero has no meaning.

11.3.3 Cultural Inhibition Error

This is an error committed by the student due to past experience, which prevent that student from responding correctly to new situation. For instance, a pupil in primary two was taught by their basic science teacher that half is rabi(using Vernacular Hausa) and rabi is part of whole. In later, years this pupil was given a diagram to identify half. Student looking shaded area in figure and concluded that it is half, which is wrong.

11.3.4 Pragmatic Error

It is an error committed by student through individual bias, emphasis or connotations placed on the symbol by individual learner of mathematics. For example, if a student has understood that the equation $y=ax^2+bx+c$ is quadratic equation, whenever given $q=py^2+c$, the student will no longer see that it is also quadratic equation becausesince the teacher mainly used an equation in x.

11.3.5 Understanding Error

It is an error committed by many students given wrong interpretation to word problems. Example: The mathematics teacher asked question that there are three times many pupils (P) as there are teachers (T). Put down this statement in correct mathematics expression. This problem was forwarded to ten students in 100 level. Majority of the students obtained the result as 3P=T, only two students responded correctly as P=3T

11.3.6 Premature Approximation Error

This is an error committed by learners of mathematics where a student approximates a value obtained in a part of problem and used the approximated value in working other parts of the given problem. For example, given that the radius of the earth is 7400 km and pie=3.142 with two points P(50^0N, 40^0W) and Q(50^0N,10^0 W). Compute:

i. the radius of the parallel of latitude 50^0N

ii. the distance PQ along the circle of latitude. Correct your answer to one decimal place

Solution: if the value of r is as 7400cos30 = 6408.4 or 6408, the student approximated r=6408 and work out arc PQ=30/360×2×3×3.142×6408. This provided a wrong answer as 10,067.0 but if use correct value as 6408.4, the answer will read as10068.2

11.3.7 Mechanistic Understanding Error

This is type of error that can be committed by student for not enable to transfer knowledge to another mathematics process. For instance, student has knowledge of sum of square difference as X^2-Y^2= (X+Y) (X-Y), when given a problem as 225^2-15^2, the student went on to use long multiplication thereby committed error through long multiplication.

11.3.8 Misreading Error

This is an error committed through wrong copying of problem given. For example, 5^3+ 5^{-4}- 5^{-4}

A student in copying the problem as: 5^3+ 5^4- 5^4 the negative power, -4 was copied as 4

11.4 Semantic and Syntactic Difficult

Semantic and syntactic difficult that affect teaching and learning of mathematics will be discussed on Directed numbers, Number and Numeration, Geometry and Algebra.

According to Inekwe(2019) pointed out that communication by many teachers of mathematics for a signed number, example is -8"minus 8" where 8 is not being subtracted from any number. Naturally speaking, the statement is not proper since you can't subtract things where nothing exists. But 8 – 4 is rightly reads "8 minus 4" which is naturally possible. Hence this is a syntactic problem. Another example on number line zero is demarcation: -3, -2, -1, 0, 1, 2, 3 between negative and positive integers as shown above. hardly do we refer to the numbers right of zero as plus 1, plus 2, plus 3, but integers left of zero are refer to minus 1, minus 2, minus 3.Which are incorrect? The difficulty encountered by many students in directed numbers may not detach from this semantic problem being communicated to them.

"Zero" is refers to non-positive and non-negative integer but being defined by some mathematics teachers as nothing and that in build in the mind of young immature ones on shaky mathematical foundation. And when such errors are not checked and corrected the receivers may pass it on to another generation and it cycle continue.

Other semantic difficulties in geometry and algebra are a plane is wrongly defined as a flat surface rather than a 2-demensional surface without thickness, while a straight line to some is the shortest distance between two points rather than a set of points which continue in opposite directions indefinitely. A line segment is a set of points with two end points and a ray is the set of points with one fixed points. Because some of mathematics teacher gives unclear classification between the definition of line segment and ray. Some defined "cycle" as space between two rays. A triangle is incorrectly refers to a figure bounded by 3 lines rather than a plane figure bounded by 3 line segments.

Algebraically too, instead of if X = 5 some say when X = 5 whereas when refers to time. Logically, speaking there is never a time when X will be equal to 5, being two different entities are alphabetic and the other numeric. Instead of removing and applying the bracket, mathematics teachers commonly communicate open and close the bracket. Also some mathematics teachers say X=3 as X is equal to 3 rather than X equal 3. All these are syntactic errors that affects majority of mathematics teachers and students alike. The phrase cross multiply a/b =x/y to obtain ay = bx, instead of multiply both side by the L.C.M of the denominators to obtain ay = bx. Hence teachers of mathematics are advised rather to avoid such mechanical terms like cross-multiply, which lead to erroneous application especially at the primary level.

Phonologically 2008 is pronounced by many, as 2008 as in book instead of 2 zero 8, symbol O (numeric zero) is mixed up with letter O (alphabetic). Also, the write up of numeric zero (0) is spherical while that of alphabet is circle in nature (O). Instead of transfer 1 to the tens column many say carry 1 and write down 5 as in case of 7 + 5 = 15. It is "3/7" is read as 3 over 7 instead of 7 divide 3. All these are linguistic problems that affect meaning of concepts and constructions of mathematical ideas adversely. Therefore, it becomes necessary to take steps to minimize these problems.

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Student Activity

1. What type of error involved in the following solutions?
   a. $5^2=10$
   b. $6+5(a-3) =11(a-3)$
   c. $X+X=X^2$
   2. Four families have respectively 5, 4 and 8 children in their houses. What is the average
   number of children per family? Solution = 5+4+8/3=17/3=5 2/3 what kind of error committed in this answer.
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   Solution: P = 4P/hr, = 4P = Q therefore Average speed = $P + P^2 / 4$
   Identify the error in this solution.
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11.5 Summary

The content of this chapter is summarized as follows:

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CHAPTER TWELVE

RESEARCH CONCEPTS

12.1 Objectives

At the end of this chapter, you should be able to:

i. Define the terms research and mathematics education research
ii. Explain basic steps that involves in research process
iii. Discuss the characteristics of research
iv. Explain the importance of mathematics education research; and
v. List types of educational research

12.2 Introduction

Effective education is based on a solid foundation of fresh information. We can develop new information and/or alter current knowledge through study. To be effective, teachers must evaluate their teaching methods and modify them if they are ineffective. In addition, empirical research informs the majority of global education system policy. Teachers and students must thus possess strong research abilities.

12.3 Definition of Research

Research is not concerned with problems for which a solution is clear. Nonetheless, evidence indicates the following: - Empirical research of a problem; an inquiry into the unknown; an endeavor to give a comprehensive or partial solution to a problem; a systematic method of problem-solving; a search for new knowledge; a process of developing new ideas or information.

Mamman (2007) defines research as the systematic exploration of a topic with the intent of advancing knowledge. Research is the application of the scientific method to the methodical investigation of issues. Research may also be defined as a thorough examination or inquiry conducted in the pursuit of new facts in any field of knowledge.

12.4 Types of Research

There are essentially two sorts of research: basic or fundamental research and applied or action research (Best & Kahn, 2009). Generally, fundamental research is conducted in a laboratory or other sterile setting, and occasionally with animals. This form of study, which typically has no immediate or planned application, may eventually lead to more applied research.

Applied research has many of the same characteristics as basic research, including the use of sampling techniques and subsequent inferences about the target population.

However, its purpose is to improve a product or a process by putting theoretical concepts to the test in real-world problem situations. According to (Best & Kahn, 2009), any attempt to identify categories of educational research is problematic, yet the majority of educational research is applied or action research (Mishra, 2005 and Koshy, 2005). Following is a partial list of educational research types:

Case study; Evaluation Research; Historical Research; Descriptive Research

Correlation, longitudinal, causal comparative, and quasi-experimental research.

12.5 Research in Mathematics Education

It is the methodical process of conducting well-planned inquiries with the goal of resolving a recognized problem in mathematics education. According to Sambo (2008), mathematics educational research is a systematic process of inquiry and study of difficulties and challenges in the field of mathematics education with the primary purpose of identifying potential answers. Therefore, mathematics educational research is any systematic process of inquiry, analysis, and discovery that works with all concerns pertaining to students, instructors, the teaching and learning processes, and their consequences.

12.6 Characteristics of Research in Mathematics Education

Mathematics education research is a scientific endeavor. It is scientific activity because it contains objectivity, verifiability, and dependability or repeatability, among other characteristics. Based on this, let us examine some aspects of mathematics education research. These include:

i. Research in Mathematics Education is Systematic: This implies that all established norms must be adhered to in a systematic and logical manner.

ii. Research in Mathematics Education is Objective, in the sense that the researcher must publish the findings without prejudice, preconceptions, or societal influence. The researcher should not allow personal interests to interfere with the study process or the analysis of the data gathered.

iii. Research in Mathematics Education is Reproducible: The researcher or any other interested researcher can check the results. It is possible to re-administer a similar instrument to the individuals or group; this might help to support the validity of the findings.

iv. Study in Mathematics Education is Quantifiable: It necessitates that accurate observation, description, and data collected for research be measurable and verified against hypotheses.

Research in Mathematics Education is Characterized by Patience and Demands Fortitude This indicates that research work is characterized by calm, unhurried effort and requires courage.

Research Report in Mathematics Education is Precise: This indicates that the researcher must be meticulous in their report and documentation. In drafting a report, it is beneficial to utilize straightforward language that facilitates comprehension.

12.7 Importance of Research in Mathematics Education

Research is now an integral component of mathematical instruction. Consequently, the following might be considered some of the significance of educational research:

§ It gives instruction in both problem-solving and leadership.

§ Provide adequate explanations for the various failure causes.

§ It enhances comprehension of educational phenomena.

§ Instrumentation and design development for measuring educational ideas.

§ It aids in educational decision-making.

§ It contributes to advancing understanding.

§ It offers the area of education with a framework within which the process and output of the educational system may be questioned and justified critically.

§ Provides instructors with an understanding of how to manipulate factors so that they may raise their students' or pupils' chances of success or give better strategies that will enhance teaching and learning.

§ It can contribute to the literature on effective practices and assist mathematics instructors implement more effective methods in their classrooms.

§ It can give information on individuals and locations that have not been investigated previously.

§ To create and/or evaluate educational theories.

Student engagement

1. a. What exactly is research?

   b. What is research on mathematics education?

2. List any five essential mathematics education research findings
3. Describe any five qualities of an excellent mathematics education research
4. List three categories of educational research and briefly describe two of them.

12.8 Summary

In this chapter, you have learnt the definitions of research and mathematics education research. You have learnt the characteristics of good research in mathematics education and also its importance.

References

Anaekwe, M. C. (2002). *Basic research methods and statistics in education and social sciences.* Enugu: Podiks Printing and Publishing Company

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CHAPTER THIRTEEN

RESEARCH PROBLEM

13.1 Objectives

By the end of this chapter, you should be able to:

i. List and explain the basic steps in the research process.
ii. Enumerate the criteria to be consider in choosing research problem; and
iii. Discuss the sources of research problem

13.2 Introduction

The initial stage in doing educational research, particularly mathematics education research, is to identify the research problem; all efforts are directed in this direction. You must create an investigation mode to address the detected issue. In addition, this chapter will discuss criteria for issue selection, sources of research problems, and fundamental research procedures.

13.3 Fundamental Steps in the Research Methodology

Education research include investigating, describing, explaining, or predicting educational phenomena using systematic data gathering and analytic techniques. The following are fundamental phases in the research process, as stated by (Johnson & Christensen, 2004, McGowan, 2011 and Creswell, 2012).

1. Formulating a research question
2. Make a forecast that, if validated, will address the issue.
3. Reviewing the literature 4. Establishing a research objective
5. Collecting relevant data
6. Data analysis and interpretation
7. Research reporting and evaluation

13.4 Features of the Research Problem

Choosing a research subject or topic is the most complex and difficult aspect of a graduate research project. According to Best & Kahn (2009), the selection of a good problem or topic is always challenging, particularly for novices, and a lack of awareness of the nature of research and methodical problem-solving activity are significant factors. It might also be due to their zealous yet naive desire to tackle an important issue promptly and rapidly. When all of their submitted themes are rejected by their supervisors on the grounds that they are not excellent topics or researchable, many students become dissatisfied.

A research problem or topic is the subject one wishes to analyze, study, or investigate descriptively or experimentally. This is the rationale for doing the research. The following can serve as criteria for selecting a proposed research problem or subject, since students should not anticipate their supervisors or advisers to provide them a topic. These include contribution to current knowledge, researchability, appropriateness, practicability, and cost-effectiveness.

i. *Contribution to knowledge:* A research topic must not be of the routine kind, that is, the results of the problem must not be already known. The challenge should be one whose answer will contribute to the body's structured learning and understanding. It should result in new or increased knowledge in schooling.

ii. *Researchable:* The issue must be one that can be investigated. To be researchable, a problem must be concerned with relation existing between two or more variables that can be described and quantified.

iii. *Appropriate:* The problem must be appropriate for the researcher in question. The research suitability entails the following: -

a. The researcher must have a genuine interest in the issue at hand.

b. The problem must be within one's area of expertise and experience. Existing ideas, concepts, and established facts must be understood.

c. The problem must be practical, especially as it pertains to the accessibility and availability of the necessary data for the investigation. The variables included in the research should be objectively and operationally definable.

d. The issue must be one that can be researched and resolved within the allocated time.

e. It builds on prior work and provides something novel. It has the ability to offer future study areas.

f. Less Expensive: When selecting a study topic, you should examine the availability of the necessary human, material, and financial resources. Avoid problems involving huge sources that you cannot accommodate.

g. Save Time: When selecting a research issue, you consider the time constraint so that a realistic answer is possible. The following requirements must be met by a research subject in order to justify its objectives:

13.5 Origins of the Research Problem

Aliyu (2017) and Ahmad (2020) outlined the numerous sources from which research themes and issues might be derived. These consist of:

- Personal Experience
- Internet Source
- Government Policy or Publications
- Prior Literature
- Theory

§ Replication

Student Exercise

1. Explain the criteria for selecting a study topic.
2. Outline the seven fundamental phases in research process.
3. Describe any five sources that might be used to acquire research problem

13.6 Summary

In this chapter you leant basic steps in research process, conditions to consider in selecting research problem and sources of getting research topics.

References

Anaekwe, M. C. (2002). *Basic research methods and statistics in education and social sciences.*Enugu: Podiks Printing and Publishing Company

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CHAPTER FOURTEEN

RESEARCH PROCESS

14.1 Objectives

At the end of this chapter, you should be able to:

i. Explain the background of the study;
ii. Discuss the statement of the problem;
iii. Explain the objective of the study;
iv. Describe the significance of the study
v. State the research question
vi. Formulate the research hypotheses
vii. State the basic assumption

14.2 Introduction

In this chapter, you will read about how to explain the background of the study, statement of the problem and other sub-headings in chapter one of research report.

14.3 Background of the Study or Introduction

This section is intended to introduce the research problem. The introduction contextualizes the research topic within a particular discipline. It is usually prudent for the author not to presume that the reader has a comprehensive understanding of the study's topic. The background material should describe the problem's origin, its breadth, and the extent to which past studies have effectively researched the issue, highlighting any gaps that your study seeks to fill.

It is the responsibility of the author to introduce the reader to the issue area in as succinct a manner as feasible. By citing reliable sources, the researcher may demonstrate that the examination of the issue area had been inadequate, insufficient, or nonexistent. The researcher must provide a clear picture of the circumstances and events surrounding the inquiry, including their impact on the topic being studied. All facts required to comprehend the problem area under examination should be stated succinctly and rationally. All factors included in the topic of study should be briefly addressed and connected.

Example; "Effects of Scaffolding Learning Strategy on Senior Secondary School Students' Achievement and Attitude in Solid Geometry" For the topic above, issues to be discussed in the background of the study should include:

i. The importance of mathematics in the overall development of human activities, including its role in the development of science and information technology;
ii. The significance of solid geometry; and

iii. These factors may include the inadequate background of the pupils, the lack of subject-matter expertise of the teachers, the substandard educational atmosphere, and the ineffective teaching methods.

iv. Discuss briefly the attempts made by previous students to tackle this problem and why the researcher believes his work will help solve or decrease the issue.

v. Discuss briefly the notion of scaffolding strategy and the theoretical foundation upon which it was constructed.

vi. Discuss attitude and how it affects mathematics learning briefly.

14.4 Statement of the Issue

The problem should be stated in straightforward language. The problem statement might be either declarative or interrogative. It explains why there is a deficiency or an unacceptable situation in the specific area of research. It is crucial for the researcher to demonstrate how the solution to the problem or the answer to the question might enhance teaching and learning. In other words, the researcher must justify why the planned study is worth the time, effort, and price (Best & Kahn, 2009). A proper formulation of the problem must specify precisely what is to be determined or addressed and limit the scope of the research to a single subject. The problem should neither be too broad nor too specific. If the claimed problem is very broad and imprecise, there is no clear indication of the research's intended path. On the other hand, the problem must not be so limited as to be meaningless and trivial. The search problem is mentioned in a few phrases. According to (Creswell, 2012), widely used statements for the description of problem include: 'the objective of the study will be...', 'this study will be done to...', and 'the problem to be studied by this study is...'.

Example; "The Effects of a Scaffolding Strategy on the Achievement and Attitude of Senior Secondary School Students in Solid Geometry"

According to the preceding debates and the findings of researchers, one of the causes of poor performance is an overemphasis on syllabus covering at the price of meaningful acquisition of mathematics ideas (Onwuka & Moseri, 2011). The lecture technique is still widely employed in Nigerian mathematics classrooms, despite its evident and significant limitations (Kankia, 2008). National Council of Teachers of Mathematics (NCTM, 2014) has advocated learning mathematics with comprehension as the vision for how school mathematics might be taught (Abdullahi, 2010). The purpose of this study is to contribute to the body of knowledge by investigating the effect of scaffolding teaching tactics on the accomplishment and attitude of senior secondary school one (SS1) students in trigonometry.

14.5 The Purpose of the Study

The objective, purpose, or goal of the study describes the primary steps the researcher wants to take to provide a solution to the problem indicated in the problem statement. Typically, the objective of the study serves as a guide for the planned research topics. The study should specify the desired outcomes, ensuring that a solution to the identified problem is in place by the time these objectives are met. Therefore, the objective of the study must be explicit and unambiguous for the researcher. The entire research effort hinges on the study's goal. It is a good habit to list the tasks that will be performed for the goal of the research.

Example: "The Effects of Scaffolding on the Achievement and Attitude of Senior Secondary School Students in Solid Geometry"

The primary objective of this study was to examine the impact of scaffolding skills on the success and attitude of senior secondary students about a particular notion of solid geometry. Specifically, the study aimed to:

i. determine whether there are variations in students' performance in solid geometry when taught utilizing a scaffolding technique.

ii. determine if there are variations in the accomplishment of male and female students in solid geometry when taught utilizing the scaffolding technique.

iii. determine if students taught utilizing a scaffolding method will experience a change in their attitudes.

14.6 Research Problems

These are built and organized questions aimed to assist the researcher to a solution for the identified problem. By posing the questions, the researcher seems to imply that solutions to the problems can be found if these questions can be answered. Agee (2009) suggested that an initial generating question might be posed, followed by the formulation of subordinate inquiries. Depending on the subject of the overall inquiry, several sub-questions may be posed. If this is the case, the researcher must guarantee that the questions positively correspond with the goal of the study, or that they are congruent.

Example: "Effects of Scaffolding Strategy on the Achievement and Attitude of Senior Secondary School Students in Solid Geometry"

On the basis of the mentioned objectives, purposes, or goals, the following research questions can be formulated:

i. What are the mean performance scores in solid geometry for SS1 students taught using the scaffolding technique vs those taught using the lecture method?

ii. Will there be any discrepancies between male and female performance in solid geometry when the scaffolding technique is utilized?

Is there a difference in the attitudes of male and female students who use the scaffolding strategy?

14.7 Scientific Hypotheses

In investigations that need the manipulation and control of variables and those that seek to demonstrate a cause-and-effect connection between variables, the development of nontrivial hypotheses becomes vital. A hypothesis is a reasonable response to the investigational inquiries. According to Tashakkori and Creswell (2007), hypotheses are the researcher's predictions on the predicted relationships between variables. Notably, research hypotheses should be consistent with the study topic and given in null form, i.e. "no significant difference" or "no link.""

Example; "Effects of Scaffolding Strategy on the Achievement and Attitude of Senior Secondary School Students in Solid Geometry"

On the basis of the aforementioned research questions, the following research hypotheses might be formulated:

i. There is no significant difference in the mean performance scores of SSI students who were taught solid geometry using the scaffolding technique and those who were taught using the lecture method.

ii. There is no significant difference between male and female performance in solid geometry when scaffolding method is employed.

There is no discernible difference in the attitudes of male and female students taught using the scaffolding approach.

14.8 Importance of the Research

Every research endeavor is performed with the intention of advancing education-related knowledge. The inquiry may have been conducted to address a practical issue or to address a gap in educational theory. The researcher must demonstrate the significance of the proposed study and the impact it would make if carried out. The researcher describes who will profit from his study and how they will do so.

14.9 Range or Boundaries of the Study

This section should outline the study's limits in terms of its subject matter. Each research conducted in a field represents a small portion of a vast field. Thus, in this part, the researcher should exhibit understanding of the regions or dimensions of the issue and then precisely describe the area he/she want to address in the study. The researcher should specify the boundaries or limits of the investigation. It helps to place the study in

its right context, allowing the researcher to determine the precise component of the topic he wishes to investigate and the scope of the investigation. Delimitation is performed to make the study viable, practical, conductable, and generalizable. For instance, delimitations are created by providing the time, the location of the population, and the sampling site.

14.10 Fundamental Hypothesis of the Study

Assumptions are statements of what the researcher thinks to be facts but cannot verify. A researcher may assume that the participants, i.e., the SSI students in the study, have studied the mathematical content they are expected to cover in their JSS. Likewise, the instructors in the sampled schools are qualified and competent to instruct at those levels.

Student Activity

For the topic "**The Effectsof Improvised Instructional Materials on Students Performance in Solid Geometry**"

1. a. Mention any 4 things that need to be included in the background for the topic above.

b. State any two research questions and corresponding two hypotheses.

2. Mention any five possible significances of the topic above.

3. For the topic "**The Survey on Relationship Between the Attitude and Academic Achievement of N.C.E II Students of Niger State College of Education.**

a. Mention any three purposes of the research.

b. Mention any two scopes of the study.

14.11 Summary

In this chapter, background of the study was discussed as part of item containing in chapter one of research process. In this chapter also, statement of the problem, objective, scope of the study, basic assumption, including research questions and hypotheses were learnt.

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CHAPTER FIFTEEN

LITERATURE REVIEW

15.1 Objectives

By the end of this chapter, you should be able to:

i. Define what literature review is;
ii. Write literature review in any research study
iii. Outline the sources to get literature review
iv. Explain the importance of literature review

15.2 Introduction

Typically, the literature review is the second chapter of a research paper. The literature review is a crucial component of the research procedure. It motivates, informs, educates, and illuminates. It creates ideas, aids in the formulation of meaningful research questions, and facilitates the research design process. It is also essential to the writing process. A well-constructed literature review is an essential factor for establishing the credibility of a researcher. This review aims to provide the researcher with a thorough understanding of the state of the art in the field in which he or she is doing research. It informs the researcher of the regions covered, the areas yet to be explored, and the investigative approaches to utilize.

15.3 What is Literature Review?

When doing any study, you must reference the work of others to see how far they have progressed in the field. By doing so, you will learn from prior work and have a solid grasp of the subject field.

Books review entails reading literature and analyzing what others have done in the past. The primary objective is to extend what is known about the study issue and to add fresh information to what is already known. A literature review is an evaluation of research discovered in the academic literature that pertain to a person's chosen field of study (Book & Beile 2005).

15.4 Literature Review Sources

The examination of relevant literature entails the methodical identification, location, and evaluation of papers providing information pertinent to the study issue. The papers include research reports (projects, dissertations, and theses), textbooks, reviews, magazines, abstracts, yearbooks, journals, government publications, speeches, conference and seminar/workshop reports, and newspaper stories.

15.5 Literature Review Writing

An effective literature review is more than a summary of research papers and their results. Rather, it depicts the key concerns and interrelationships related with the investigated information, arguments, and topics. This type of literature review describes what has been published about the subject, how this information has been regarded by other academics, and the most significant research findings across studies. Conducting a literature review demonstrates an author's understanding of a certain subject of research, including its terminology, theories, major variables and phenomena, and technique and history (Randolph, 2009). In creating a literature review, the following must lead the researcher:

§ The tasks must be arranged in a sensible order

§ If reviewing the literature on learning theories, begin with the earliest and work your way to the most recent.

§ Try to refer back to your research questions and hypotheses so you don't stray from the subject matter

§ Use more current sources, unless they are unavailable, or the work is really relevant. If, for instance, a book was published in 2010, you should prioritize it above one published in 1990.

§ Use your expertise to connect the concept and discovery to make them relevant, rather than leaving them as separate pieces.

§ Only data pertinent to the research should be provided.

§ Cite only those statements that the original author said so clearly and succinctly that any attempt to paraphrase them might result in a loss of their original meaning.

§ Divide the literature review into sections, focusing on the variables retrieved from the study questions or hypotheses.

15.6 Significance of Literature Evaluation

The evaluation might be helpful for the following:

i. Informing the researcher on what has previously been done in the area and what needs further investigation.

ii. Presenting potential research designs and methodological approaches that may be utilized in the research investigation.

iii. Suggesting potential modifications to the research to prevent unintended complications.

iv. Identify any potential research gaps.

v. It supplies the researcher with helpful theoretical and empirical information stated by others or previous researchers on the same topic and problem he sought to examine.

vi. It guarantees that published works are not accidentally or unnecessarily duplicated.

vii. It aids the researcher in identifying a standardized instrument that may be applicable to his study.

viii. It permits the researcher to avoid the blunders of previous researchers and gain from their experiences.

ix. It promotes a deeper grasp of the investigated issue and its most significant component.

x. It gives comparison information for evaluating and interpreting one's findings.;

xi. It describes what is known in the field of research. This comprises the most significant ideas, research methods, and outcomes.

xii. It illustrates the unresolved issues in the field of research.

xiii. The information gained from the review will also assist the researcher construct good and relevant study hypotheses

xiv. It demonstrates how the present study fits inside the grand scheme of things.

xv. It highlights the similarities and contrasts between the current study and past studies.

15.7 Acknowledgment of Information Source

Other people's work that you evaluated must be recognized. That is recognizing those who perform the task. You should not claim ownership of the concepts. There are two primary ways to cite the work of other researchers in a literature review. One is direct citation, in which the actual words of the scholar are enclosed in quotation marks. For instance, Kajuru (2010) states, "Students perform better when taught mathematics utilizing a cooperative technique as opposed to the traditional way." Kajuru(2010) emphasized that students perform better when they are taught mathematics utilizing the cooperative technique as opposed to the traditional way.

Student Exercise

1. Identify four essential considerations for preparing a literature review.
2. Provide 10 reasons why you must do a literature review for your research.
3. Mention any five literature review steps.
4. Define literature review, then list any five literature sources.

15.8 Summary

In this chapter, you learnt that literature review is reading through literature and reviewing what has been done by others in the past. Also, guidelines are provided when writing a review of literature.You have seen the importance of literature review and ways to acknowledge the scholars in a text.

References

Anaekwe, M. C. (2002). *Basic research methods and statistics in education and social sciences.* Enugu: Podicks Printing and Publishing Company.

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CHAPTER SIXTEEN

POPULATION OF STUDY

16.1 Objectives

At the end of this chapter, you should be able to:

i. explain the meaning of population.
ii. define sample; and
iii. discuss the sampling techniques

16.2 Introduction

In this chapter, you will learn the concepts of population and sample in relation to research study in mathematics education, since the two concepts are fundamental importance in any research. Also, the sampling procedure used in research will be adequacy defined and discuss in this chapter.

16.3 Concept of Population

Population is a set of components, events, objects, or all members of a well-defined class of individuals who share one or more traits of interest to researchers. For example, the total number of Mathematics students at Lafiagi College of Education. A population is also described as a collection of people who share at least one trait that differentiates them from other people (Creswell, 2012). The basic goal of research is to identify universally applicable principles, yet studying the entire population in order to reach generalizations would be impracticable, if not impossible. To address this size issue, a subset of the population must be selected for real investigation. This method of reducing a huge general population to a sample is typical in educational research (Best & Kahn, 2009).

16.4 Research Sample

Not often is it feasible for a researcher to examine the entire population. When this occurs, just a subset of the population is examined. Sample refers to the fraction of the population picked for the study. Therefore, a section of the population or universe is selected for research as a representative sample of the complete population or universe. Or, a sample is a subset of the population to whom the researcher aims to generalize his or her findings. Therefore, using a sample provides the following benefits:

- § It decreases costs
- § It saves time
- § It enables the execution of large-scale investigations

§ It increases precision

16.5 Sampling Technique

Sampling is the study of a group of individuals or observations from a larger set in order to derive conclusions about the characteristics of the larger population. According to Sambo (2008), the type of sampling technique utilized in research has a direct effect on the quality of conclusions drawn from the results of the study. There are several sampling procedures, including:

(4) Systematic Sampling

16.5.1 Simple Random Sampling

This is a method for picking a sample in which each member of the population or person has an equal and independent probability of being picked. For instance, if the chair of a college's mathematics department is tasked with selecting five 200-level students with a GPA of at least 4.0 for the COWBELL scholarship, the following criteria must be met: If there are twenty such students, the H.O.D. can assign numbers 1 through 20 to each student's name on separate pieces of paper. He can then fold these numbers, place them in a bag, thoroughly mix the papers, and ask someone to select a paper from the bag. The number and name can then be noticed and recorded. The paper was returned, and the procedure was repeated until five pupils emerged.

16.5.2 Stratified Random Sampling

It is sometimes preferable to partition the population into smaller homogenous groups in order to get a more precise representation. In the first step of this procedure, the total population is split or stratified into a number of relevant strata or groups. For instance, gender, locality, socioeconomic position, religious affiliation, years of education, or departments. Individuals are then chosen at random from each stratum or unit to constitute the sample. The numbers picked from each stratum may be proportionate or equal to the proportion of the population that falls into each stratum. There are two kinds of stratified random sampling: proportional and disproportionate (Teddlie & Yu, 2007). For instance: to pick 200 students from the N.C.E 2 students of the mathematics department at the College of Education in Minnesota. The strata can include combinations like Math/Economics, Math/Geography, Math/Physics, Math/Chemistry, and Math/Special Education. 3 Cluster or Area Sampling: is a sampling technique in which the unit of selection, known as the cluster, comprises two or more individuals of the population. Cluster sampling is effective when the individuals of a population are naturally clustered, for example. If there are ten schools, ten S.S.S. III pupils from Bosso Local Government will participate in an experiment. Here are the 10 schools in the group or unit. Consider that the researcher requires two schools, one for experimental

and one for control purposes. Two schools might be selected at random to comprise his/her sample. Then allocate randomly one each to the experimental and control groups.

16.5.3 Systematic Sampling

This is a sampling procedure in which all the members of population (Sampling Units) are arranged alphabetically (or in some natural sequence or systematic fashion) and a sample is obtained by taking individuals at a fixed interval (K^{th}), K= where "N" is the population size and "n" is the desire sample size. For example: if the HOD of mathematics is asked to select 5 students whose C.G.P.A is 4 points and above for an award. If there are 20 such students, he can assign numbers to all the students, and then randomly select one number. Assume that the first number selected is "3" then others will be selected at the interval of =4. Therefore, he will select Numbers: 3, 7, 11, 15, and 19 for the award

Student Activity

1. Briefly explain the population
2. Define the word sample
3. Distinguish between population and sample
4. List three merits of sample in research study

16.6 Summary

You now understand that the total population is the collection of people, things, events, and other things that share certain characteristics. The population's subset is known as the sample. Sampling allows for the creation of samples that are really representative of the target population. The process of selecting a sample from a target population is called sampling. These methods include basic random sampling.

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CHAPTER SEVENTEEN

INSTRUMENTATION

17.1 Objectives

At the end of this chapter, you should be able to:

i. explain the concept of research instruments

ii. describe the measuring devices: paper and pencil; observation; questionnaire and interview

iii. explain the types of interviews and questionnaire

iv. mention the merits and demerits of interview and questionnaire

17.2 Introduction

Instrumentation is the process of selecting or creating measurement instruments for collecting required data in a research endeavor. Research instruments are the measuring devices created during the instrumentation process. According to (Sambo, 2008), four frequent data gathering methods in education research are: (i.) paper and pencil achievement tests; (ii) observation; (iii) interviews; and (iv) questionnaires.

17.3 Research Instrument

According to (Sambo, 2008), four frequent data gathering methods in education research are: I paper and pencil achievement tests; (ii) observation; (iii) interviews; and (iv) questionnaires.

17.3.1 Paper and Pencil Test of Achievement

It is also known as psychometric and is a typical evaluation tool used by researchers to test cognitive levels or individual growth. To determine the level of the learning that has occurred over a period of time, academic achievement tests, quantitative and verbal aptitude tests may be used. In general, tests are designed to assess a broad range of behavioral patterns, including mental capacity or intellect, accomplishment, unique abilities, aptitude, creativity, and curiosity. In achievement test have pretest, posttest and post-posttest.

17.3.2 Observation

Questionnaire is the process of systematically observing persons, events, or circumstances in order to gather information about certain characteristics of such individuals, events, or situations. These are the two sorts of observations:

Participant: - The observer is a component of the situation or environment in which the observation is conducted.

Non-participant: - the observer in this circumstance is not a participant in the situation being observed.

17.3.3 Interview

An interview is a (face-to-face or telephone) conversation that is conducted with the intent of acquiring trustworthy and valid information from one or more respondents. There are organized and unstructured interview formats.

This is a tightly structured and extremely formal interview format. Multiple responders are provided with the same questions. The respondent must select his response from a list of options presented by the interviewer. Structured interviews are more scientific than unstructured ones because the standardized technique allows for scientific generalizations to be drawn from the data collected. The rigidity of organized interviews impedes the researcher's ability to conduct sufficiently in-depth inquiries.

This sort of interview is flexible and permits the interviewee to disclose more information than the others. This interview format is neither too rigorous nor too relaxed. It is a moderate form that can generate a considerable deal of information from the interviewee. It is challenging to quantify the obtained data while using unstructured kind. The inability to compare respondent data may preclude the possibility of generalizations with broader relevance. As a result, it is rarely employed to collect data required for testing hypotheses.

Benefits of the Interview Method

Whether organized or unstructured, the Interview technique of data collecting provides the following advantages:

1. It is adaptable and relevant to a variety of issue kinds.
2. The interviewer can observe both the content and delivery of the interviewee's responses.
3. It is really beneficial for gathering personal and supplementary information, if necessary.
4. When a respondent is motivated to accept the reasons for the interview, the interview might be highly successful.

Disadvantage of Interview

The downsides of the interview technique of data collecting are as follows:

1. It is frequently challenging to generalize from unstructured interviews.
2. The interviewer's own prejudices, biases, attitudes, and beliefs may influence the outcome of the interview.

3. The procedure is time-consuming.

4. The interviewer's race, tribe, age, sex, religion, language, accent, cultural background, or socioeconomic status might be influenced by the respondents' answers. These considerations must be taken into account while selecting interviewers.

The analysis of unstructured oral interviews is difficult to measure.

Student Exercise

1. What is a research instrument?
2. List the benefits of interviewing as a data collecting strategy.
3. What is an unstructured interview?
4. What are the features and benefits of a structured interview?

17.3.4 Questionnaires

It is a form including one or more questions about a topic or problem that asks an individual to declare his sentiments, beliefs, experiences, or attitudes toward addressing the issue or finding a solution to a particular problem by filling out the form in accordance with specific instructions. They are intended to generate precise information to satisfy a specific requirement for research data on a particular issue. This definition highlights several characteristics of the questionnaire. These consist of:

1. It is a form constructed with questions or things provided to the respondent on paper rather than orally.
2. It may include one or more elements, depending on the breadth or depth of the topic about which the researcher seeks information or data.
3. The information sought should have to do with opinion, sentiment, attitude, perception, or others, as opposed to cognitive realm.
4. The information is written down by the responder or his aide, as opposed to the researcher during an interview.

Variations on Questionnaire

There are several methods to categorize the questionnaire. Here, we will categorize the questionnaire according to the required response type. On this basis, we may distinguish between two sorts of questionnaires: fixed-response and open-ended.

Fixed-Response Questionnaire

In this form of inquiry, the responder is given a number of possible replies from which he is supposed to pick the one that best expresses his opinion, sentiment, or attitude. Therefore, the respondent is not free to select and order his own replies. Below is an illustration of this type:

Example: Section A focuses on personal information, whereas Section B tries to extract the respondent's perspective on the functioning of continuous evaluation.

Instruction: Select the proper option.

1. School Type a. all-male b. all-female c. mixed
2. Respondent's Gender: a. Male b. Female
3. Respondent's Age; a. below 20years; b. 20-30years; c. 20years or older
4. Qualification a. NCE

Which of these resources for adopting C.A. in your school are available? a. Calculators b. Progression graph c. Folder

This style of questionnaire provides a number of advantages, a few of which are given below:

§ It is simple to accomplish. Since choices are presented, even respondents with little education or language difficulties can readily complete this form of survey.

§ It saves time for the responder; • Since the replies are already organized into distinct categories, coding and quantification will be simpler. This made it easier to analyze the obtained data.

These disadvantages are connected with this sort of questionnaire:

§ It is probable that the researcher will be unable to request or receive all potential replies to items.

§ This style of questionnaire does not provide the researcher with the opportunity to collect additional information from respondents.

§ It is challenging to compose this sort of questionnaire item. This requires much time and mental work.

Fixed-response questionnaires may result in what is known as response set, in which respondents provide the same response pattern regardless of whether or not the items demand a distinct response pattern.

Open-End Questionnaire

It is a questionnaire style in which no response options are supplied. He is free to choose and organize his response in whatever way he feels fit and no constraint.

Examples of open-end questionnaire are given below:

1. Types of school----------
2. Sex of Respondent----------
3. Qualification: ----------
4. List the materials that are used in your for implementing C.A:----------

Some merits are.

§ It affords the researcher the opportunity of obtaining additional information from the respondent.

§ It is easy and timesaving to construct

§ This type could not lead to response set

Demerits of open-end questionnaire are:

§ It is difficult and time consuming to complete by the respondent

§ It requires the ability of the respondent to communicate effectively

§ Coding and quantification of data are very difficult to the researcher.

Student Activity

1. Explain the meaning of a questionnaire
2. Explain what an open-end questionnaire is.
3. Provide an example of an open-end questionnaire item
4. Which of these statements is true or false?

a. The fixed-response questionnaire is easy to construct

b. It is more difficult to code and quantify responses from fixed-response questionnaire

c. The fixed-response questionnaire can be used with people of little education.

d. All the possible responses are included in fixed-responses questionnaire

5. What is an instrument in research?
6. Enumerate the merits if interview as a method of data collection.
7. What is an unstructured interview?
8. What are the characteristics and advantages of structured interview?

17.4 Validation of Instrument

A measuring instrument might be a questionnaire, an observation or interview, or a paper-and-pencil test (i.e., achievement test). The validity and reliability of a measurement equipment should be high. It should also be useable in terms of ease, appropriateness, storage capacity, and interpretability. A measuring instrument is legitimate if it measures the quality or ability it is supposed to assess properly and precisely. As a researcher, if you wish to test intellect, attitude, achievement, and aptitude, among other factors, you should consider the following: The instrument is legitimate if it accomplishes its intended function. Four primary categories of validity can be distinguished.

Face, content, criterion-related, and construct validity are examples.

Face Validity: Recognizing a mathematical performance exam based just on its appearance. Mehren and Lehmann (1975) define face validity as whether the test seems to be legitimate at first glance. This involves exhibiting the instrument to the specialists and asking them to determine its function. A face-valid instrument offers no question regarding the sort of information it wants.

It refers to the extent to which the instrument covers the realm of interest to the investigation with precision. To guarantee that the test is legitimate in terms of its content, a table of specifications must be utilized to choose test items.

Criterion-Related Validity: The amount to which an instrument corresponds with the behavior of interest or the result of interest. That is the correlation between IQ test results and academic achievement.

Developing Validity: It pertains to the most complex instrument validation procedure. In areas where information is so restricted that other validation procedures are ineffective, it is frequently employed in research. It attempts to explain what the variables in the research signify and whether the test accurately represents their true meaning.

17.5 Measurement Error

Error in measurement may result from malfunctioning instruments, inaccurate interpretations of the results collected, or inconsistency in the respondents' or examinees' behavior. These inaccuracies may be sporadic or systematic.

17.5.1 Error Systematic

When measured values diverge from the actual score, a systematic mistake might arise. Consider, for example, your office's wall clock, which is perpetually adding time or running faster than the actual time, or a standardized test that consistently reports very high marks for every student.

17.5.2 Random Errors

When measurement values diverge from the real score, random mistakes can occur as frequently in either direction. If you take the clock as an example, you will notice that it sometimes gains time and sometimes loses time. Therefore, the extent to which the instrument measures the genuine score is indicative of two crucial factors. Validity and dependability are examples.

17.6 Reliability

This is a popular term used in daily life. In mathematics education research, however, it refers to the degree to which an instrument consistently assesses what it is intended to measure. The correlation coefficient is an essential metric for measuring the level of dependability. Methods for measuring reliability include test-retest, alternate form, divided halves, and interval consistency.

17.7 Summary

In this chapter, you learned about instruments and techniques of validity, as well as methods for estimating reliability. There are three primary forms of validity, including face validity, construct validity, and criterion-related validity.

Explained was measurement inaccuracy that may result from a defective instrument. Error may be systematic or arbitrary. Reliability is the degree to which an instrument consistently measures what it is measuring. Test-retest, alternate form, split-halves, and the internal consistency approach are the techniques for measuring reliability.

Students Activity

1. a. What do you understand by the following terms?
   b. Research Instrumentation
   c. Face Validity
   d. Content Validity
2. State and discuss the characteristics of a good measuring instrument.
3. What is reliability of an instrument?
4. What are the methods of estimating reliability?
5. List two types of measurement error?
6. Explain word validity of instrument in research study

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CHAPTER EIGHTTEEN

STATISTICAL ANALYSIS IN RESEARCH

18.1 Objectives

At the end of this chapter, you should be able to:

i. define the word statistics
ii. explain the types of statistics;
iii. organize a set of data
iv. represent data graphically
v. calculate simple percentage, measure of central tendency, correlation, z-test and t-test

18.2 Introduction

Statistics is involved with the gathering, arrangement, and analysis of data; it plays a crucial part in educational decision-making and forecasting. Practically all scientific fields, including the physical, applied, and social sciences, as well as commerce, the humanities, government, industry, and education, employ statistics.

In this chapter, we will discuss the definition of statistics in research, the types of statistics, the significance of statistics in education, data organization, and statistical tools.

18.3 Research Concept of Statistics

Statistics, in the context of research, may be defined as the application of scientific methods for transforming information gathered through a systematic process into quantifiable data, which are then subjected to quantitative analysis in an effort to draw conclusions about populations based on empirical studies. In scientific, business, and mathematics education research, descriptive and inferential studies can be categorized into two major categories.

18.4 Statistical Categories

There are two primary forms of statistics: descriptive statistics and inferential statistics.

The objective of descriptive statistics is to organize and describe the features of educational variables without reaching conclusions. Mean, mode, median, standard deviation, range, percentage, and proportions are the topics covered by descriptive statistics.

Inferential statistics use descriptive statistics' features to test hypotheses and make conclusions. This statistic includes procedures such as the t-test, f-test, and ANOVAs.

18.5 Value of Education Statistics

In the following ways, statistical analyses are extremely beneficial to mathematics education researchers:

§ Statistical analyses assist academics in quantifying the properties of variables in order to draw relevant findings on educational factors.

§ Statistical analyses aid researchers in statistically comparing factors in order to get appropriate conclusions on the investigated educational variables.

§ Statistical analyses derived from accurate conclusions about a population based on empirical study aid researchers in drawing such judgments.

§ Researchers in mathematics education who seek to read, evaluate, and interpret study results for publication in textbooks and academic publications benefit from knowledge of statistical analyses.

18.6 Administration of Data

Data collected to make such scores more relevant or manageable for computation must be written in sequential sequence and rounded to the appropriate number of significant figures or decimal places.

18.6.1 Sequencing

The data may be arranged in ascending or descending order by magnitude.

For example, seven students have the following mathematics test scores: 9,7,11,6,10,8,12

By ascending position: 6,7,8,9,10,11,12

While in decreasing sequence as follows: 12, 11, 10, 9, 8, 7, 6, and 6

Note that when collected data consist of names, they might be organized alphabetically.

However, if the data consists of things, animals, events, etc., you may organize it according to its categories or groupings.

18.6.2 Significant Figures

The first non-zero digit seen, starting from the left, is used to determine the significant digits. When the requisite number of significant figures has been subtracted, the remaining digits are eliminated according to the following rule:

If the first digit to be eliminated is a 5 or more, the final significant digit is raised by 1.

For example:

1. Approximate 6.2543 to two, three, significant figures

6.2543 = 6.3 to 2 sf

6.2543 = 6.25 to 3 sf

2. Approximate 0.000856 to one sf, two sf.

i. 0.000856The zero before the real number is not a significant figure. The rounding off will take place after the first real number.

0.000856 = 0008 to 1 sf

ii. 0.000856 = 0.00086 to 2 sf.

Note that the number zero is only significant if only situated after any non-zero real number in the whole number part e.g., 5409, the zero here is significant, but in 0.058, 17.60 and 0.000067 are zeros that are not significant.

18.6.3 Decimal Point

Decimal point is sometimes referred to decimal place and abbreviated to **dp**. These are counted to the Right of the decimal point and contained the same rules of rounding off in significant figures. For example,

Round off each of the numbers to

i. one decimal place

ii. two decimal places

a) 0.006

b) 7.6020

a) 0.006 = 0.0 to 1 dp

= 0.01 to 2 dp

b) 7.6020 = 7.6 to 1 dp

= 7.60 to 2 dp

18.6.4 Frequency Distribution Table

When the researcher wants to carry out the analysis, you needto summaries and organize the data in a frequency table. This facilitates the analysis of the data collected. A frequency table shows how many times each score in a distribution occurred. It consists basically of three columns – score column, tally column and frequency column. More columns could be added depending on what other information is required.

To construct a frequency distribution table, there are three basic steps.

(i) Listing the different scores in the distribution under the score column beginning with the highest scores on top.

(ii) Tallying the scores. Tallying of scores involves placing a stroke against a score each time the score occurs in the distribution and strokes are arranged in bundles of five for easy counting. Example 3, III, 4, IIII, 5, IIII, 6, IIII, I

(iii) Determining the number of times each score occurred. The number of strokes at each score is counted and written down as frequency of that particular score.

We now illustrate these steps with data collected from 12 students on their interest towards mathematics. 9, 8, 6, 6, 8, 8, 9, 9, 10, 12, 12, 12, 6, 6, 10, 6, 9, 5

The highest score in the distribution is 12 while the lowest score is 5.

Table 1: Frequency Distribution of Data in Mathematics Interest.

Score X	Tally	Frequency
12	III	3
10	II	2
9	IIII	4
8	III	3
7	I	1
6	IIII I	6
5	I	1
		20

18.6.5 Grouped Frequency Distribution

There are occasions when there are so many scores that it is essential to combine various scores. A class interval is made up of a set of score values.

Example

Present the scores below in a grouped frequency table.

55, 62, 60, 50, 52, 58, 55, 9, 59, 53, 52, 33, 48, 65, 60, 36, 68, 45, 62, 59, 60, 33, 40, 61, 38

60, 51, 55, 68, 55, 47, 39, 58, 52, 47, 42, 48, 55, 48, 46, 55, 51, 58, 65, 52, 35, 54, 55, 52, 56, 46, 65, 53, 34, 48, 50, 3

Before grouping the scores, the researcher has to determine the class size to use. To do this, the following procedure should be adopted.

(i) Find the range from the score; we have 60 – 33 = 27

(ii) Determine the number of groups. It is customary to have between 10 and 15

(iii) Divide the range by the number 10 e.g. 27 ÷ 10 = 3 (approximate). It is approximate to the nearest odd number because it is good to use odd number.

(iv) Draw a table and tally the scores according to groups

Table: Grouped Frequency Distribution Table

S/N	Class Interval	Tally	Frequency
1	66 – 68	II	2
2	63 – 65	III	3
3	60 – 62	IIII II	7
4	57 – 59	IIII	5

6	51 – 53	IIII III	8
7	48 – 50	IIII	5
8	45 – 47	IIII	5
9	42 – 44	II	2
10	39 – 41	III	3
11	36 – 38	II	2
12	33 – 35	IIII	4
			56

In order to arrange the scores into class intervals, the class size must be determined. For informational correctness, you should guarantee that the number of class intervals is between 10 and 20 when deciding class size. Also desired are calculations for class sizes of 2, 3, 5, and 7. Understanding interval, class size, class limitations, and class borders are fundamental.

A class interval is a set of scores in which the number of scores within each group is uniform. It has both the lowest and greatest ratings of the tiny group. Class size refers to the number of scores contained within a class interval. Each of the two highest and lowest scores in a class interval is referred to as a class limit. The lowest score is the limit for the lower class, while the highest score is the limit for the higher class. For instance, the class interval 66–68 has a class size of 3. The limit for the lower class is 66, while the higher-class limit is 68.

Any pair of successive class intervals is separated. To determine the class border, you deduct 0.5 from the minimum class limit and add 0.5 to the maximum class limit. Example 66–68 will be shown as 65.5–68.5

Example of class limits and class boundaries

Class Limit	Class Boundaries
2 – 4	1.5 – 4.5
5 – 7	4.5 – 7.5
8 – 10	7.5 – 10.5
11 – 13	10.5 – 13.5

18.7 Graphical Representation of Data

Following, a researcher used frequency distribution tables to arrange the data. Graphs are sometimes used to arrange data. The term "graphical representations" refers to this. Bar charts, histograms, frequency polygons, pie charts, and ogives come in various varieties. The bar chart and the histogram are the two types of data representation that are most frequently utilized in studies on mathematics education.

Bar Chart

A bar chart, also known as a bar diagram or a bar graph, consists of bars that stand out from one another. This demonstrates that the measurement scales are discrete rather than continuous. Bar graphs depict the relative frequency of instances within each group. It has vertical and horizontal axes. The vertical axis is termed the ordinate, while the horizontal axis is known as the abscissa. The bars may be vertical or horizontal, and the line bars or columns are of equal height; however, the height changes based on the proportion of the data.

Using a pie chart, a researcher interested in evaluating the performance of students in different departments, for instance, might exhibit the data. The examination officer of the school of science, for instance, displays the performance of students in several school departments as follows: Integrated Science = 60, Biology = 55, Chemistry = 30, Computer Science = 40, Mathematics = 25, Physics = 20.

The data can be represented in a bar chart by the following, these steps:

Step 1: Choose a convenient scale to draw the two axes (vertical and horizontal).

Step 2: Make out the height of each section based on the chosen scale.

Step 3: Draw out the bars of each to represent the height.

Histogram

A histogram is a graphical depiction of data points arranged into user-specified ranges, comparable to a bar graph. The histogram consists of a collection of rectangles with bases and intervals between class borders. Each rectangle bar represents data, and each rectangle is contiguous to its neighbors. It is created by charting the frequencies vs the class borders of the matching class interval. It is mostly used to represent continuous data below.

To create a histogram, the following steps must be taken.

Step 1: Compose frequency distribution table having the class interval, adjusted class boundaries and the frequencies.

Step 2: Choose suitable scales for both axes and draw vertical and horizontal axes.

Step 3: Label the axis based on the chosen scales.

Step 4: Draw rectangular bars on each boundary with the height corresponding to the

frequencies.

Step 5: Draw arrows to indicate what is on the vertical and horizontal axis.

To draw the histogram, you then adopt the listed procedures 1 – 5.

To illustrate the histogram scores of 80 students in MAT 224 test at end of 2nd semester exams in a certain College of Education as follows:

Scores	**Number of Students**

53 – 55	11
56 – 58	14
59 – 61	10
62 – 64	8
65 – 67	7
68 – 70	6
71 – 73	9
74 – 76	5
77 – 79	5

Table: MAT225 Test Score for 2nd Semester Exams

S/N	Class Interval	Class Boundary	Frequency
1.	50 – 52	49.5 – 51.5	5
2.	53 – 55	51.5 – 55.5	11
3.	56 – 58	55.5 – 58.5	14
4.	59 – 61	58.5 – 61.5	10
5.	62 – 64	61.5 – 64.5	8
6.	65 – 67	64.5 – 67.5	7
7.	68 – 70	67.5 – 70.5	6
8.	71 – 73	70.5 – 73.5	9
9.	74 – 76	73.5 – 76.5	5
10.	77 – 79	76.5 – 79.5	5
	TOTAL		**80**

Frequency Polygon

This is a type of graph of a frequency distribution which is obtained by plotting the class frequencies against the class marks. It is polygon because the mid-point of the tops of the rectangles in the histogram are connected.

A frequency polygon is constructed by the following the procedure bellow:

Step 1: Draw both axes (i.e., vertical, and horizontal).

Step 2: Mark out the frequencies along the vertical axis and the mid-points of class intervals on the horizontal axis.

Step 3: Plot the frequency of each class interval at the appropriates height as a point above the mid-point of interval.

Step 4: Join these points with straight lines.

Step 5: Connect the first and last dots with the horizontal axis at the mid-point before the

first dot and the one after the last dot.

Use the data above to present a frequency polygon by adopted step 1 – 5.

frequency polygon of data in table

Students' Activity

1. A newly admitted mathematics students of ABU Zaria spent a total of N60,000.00 with the following details:

- Tuition fees = N10,000.00
- Game fees = 5,000.00
- Clinic fee = N2,000.00
- Course materials = N15,000.00
- Accommodation = N10,000.00
- Stationeries = N5,000.00
- Feeding = N10,000.00
- Notebooks = N3,000.00

Construct these exercises in pie chart.

2. Consider the following scores obtained by a researcher by administering 40 students in post-test.

30	25	54	50	12	5	18	40
21	25	55	13	40	3	46	3
23	21	34	49	18	8	48	18
39	27	37	30	15	5	23	46
21	21	33	35	5	8	45	38

a) Prepare a frequency distribution table for the data using class size of 5.

b) Draw a bar chart.

c) Draw a Histogram.

d) Draw a frequency polygon.

18.8 Simple Percentage

This is the simplest of all the statistical methods used in analysis of data. What is usually done is to translate frequency counts into percentages. It is useful in making general statements about a given situation and for comparing different parts of a whole or a given situation. This is because; the percentage treats the groups as formula for calculating the percentage is

= x 100

To illustrate the calculation of the percentage, let us suppose that a researcher interested in comparing the performance of the students in the different science subjects

in the SSCE.

Numbers of students who enrolled and passed in the science subjects in SSCE are given below:

Subject	Nos Admitted	Nos. Passed
Biology	8,500	4,500
Chemistry	2,320	1,800
Physics	1,800	1,500
Agric. Sci.	9000	3,000

The percentage of students who passed in various subjects is calculated as follows:

% passed in Biology = x = 52.94%

% passed in Chemistry = x = 77.59%

% passed in Physics = x = 83.3%

% passed in Agric. Sci.= x = 33.33%

If you use the number or frequency of students who passed Biology (4500) and the number who passed Chemistry (1800) in comparing performance in the subjects, you may be tempted to conclude that performance in Biology is better than performance in Chemistry. However, if you use the percentage of students passing in the two subjects (52.94% for Biology and 77.59% in Chemistry), you will see that performance in Chemistry appears to be better than performance in Biology.

Student Activity

1. In a research to compare drop-out rates junior secondary school in three towns, a researcher collected the following data.

Town	No.Enrolled	No. Dropped out
Bida	6,800	400
Edozigi	10,200	2,000
Pati	5,000	900

(a) What is the percentage drop-out in each of the three towns?

(b) What percentage of the total number of students in the three towns dropped out?

18.9 Measure of Central Tendency

The mean, median and mode are measure of central tendency or measures of location. There are very useful statistics for reporting research findings. They give you information about the characteristics of an average or typical member of a group. The characteristics may be performance, interest, attitude etc.

18.9.1 The Mean

This is also known as the arithmetic average. It is the sum of a distribution's scores divided by the entire number of scores.

The algorithm is where is the sum of scores, N is the total number of scores, is the mean.

Example 1:

The scores of ten students in a test are as follows: 40, 55, 60, 30, 50, 48, 70, 85, 72, and 65. Find the mean.

= 40 + 55 + 60+ 30 + 50 + 48 + 70 + 85 + 72 +65 = 575

N = 10 items (scores)

= = = 57.5

18.9.2 The Median

The middle score, or median, divides the total scores into two equally-sized halves. The scores must be sorted in an ordering, either ascending or decreasing, to obtain the median.

Example

Find the median of the sets of scores

(a) 9, 7, 15, 10, 11, 8, 2, 4, 3

(b) 5, 9, 8, 7, 3, 2, 4, 6, 5, 8

In example (a) We have 2, 4, 5, 7, 8, 9, 10, 11, 15. By counting, the middle number, which is 8, is the median.

In example (b), you will notice that the number is even. You will therefore arrange in order by counting, the two middle numbers are taken, added and divided by two.

We have: 2, 3, 4, 5, 5, 6, 7, 8, 8, 9. The median is = = 5.5

18.9.3 The Mode

This is the score or scores that appear the most frequently in a distribution. Inspection makes this determination simple. But in some distribution, you may have two modes. This is referred to as bimodal, while multi-model refers to any distribution that has more than two modes.

Example

Find the mode in the distribution: 20, 30, 21, 45, 30, 25, 33, 35, 30, 22, 29, 30.

By inspection, you will see that 30 appeared 4 times. It is the mode.

Student Activity

1. What is the meaning of the following terms; mean, median and mode

2. Compute the mean, median and mode of the scores below:

   10, 7, 8, 9, 6, 9, 3, 2, 9, 5, 1, 2, 5, 0, 5, 7, 8, 5, 6, 4, 5

3. Present the scores below in a grouped frequency table

The researcher obtained scores after testing 65 students in Mathematics:

33	26	29	41	36	36	26	41	37	19	37	29
44	37	30	31	29	41	28	23	44	29	46	51
49	55	60	29	37	36	34	26	30	28	23	25
47	50	48	65	38	39	26	41	34	25	40	30
24	50	48	47	56	33	38	37	37	30	41	30
41	24	51	49	50							

4. Distinguish clearly between the following pairs of concepts giving a suitable example in each case.

   (a) Descriptive and Inferential Statistics
   (b) Class interval and class size
   (c) Frequency polygon and histogram
   (d) Bar graph and pie chart
   (e) Class mark and class boundaries
   (f) Multi-modal and Bimodal

5. Consider the following scores obtained by a researcher by testing 40 students in geometry concepts in Mathematics

38	5	45	40	13	33	21	30
46	8	23	12	50	37	27	39
40	8	48	15	49	34	25	21
18	3	46	18	35	55	21	21
3	5	18	5	30	54	25	23

(a) Using a class size of 5, prepare a frequency distribution table for the data

(b) Using your frequency table (i) Draw bar graph (ii) Draw a histogram

18.10 The Concept of Correlation

The degree of link between two variables is referred to as correlation. The correlation coefficient is a metric that expresses how strongly two variables are related. The range of values ranges from -1 to 1. Accordingly, a correlation coefficient of -1 denotes a perfect negative connection, a correlation coefficient of +1 denotes a perfect positive relationship, and a correlation coefficient of 0 denotes the absence of a relationship. A researcher can establish if variations in one set of scores are the result of variations in another set of scores using correlation. Determining the magnitude of this deviation will

also be helpful. Plot two sets of scores, X and Y, on the Cartesian coordinate plane once you have a scatter diagram. This produces a link that is either positive, negative, or neutral.

This suggests that those who perform well on one variable also perform well on the second. Additionally, it implies that those who perform poorly in one variable also perform poorly in the other variables.

Contrary to a positive connection, a negative relationship is the reverse.

This indicates that there is no link.

18.10.1 Correlation Coefficient Computation

There are several correlation coefficient calculation techniques. The Pearson Product Moment Correlation Method and the Spearman Rank-order Correlation Method are two of these techniques. The PPMC is the most extensively utilized and is named after its creator, Karl Pearson. There are two primary methods for computing the PPMC (r).

The first is the deviations from the mean method, while the second is the raw scores method. Let us examine each of them individually.

Deviation from the Mean is given by using:

or

Where x = x - , y = y -

Example 5: The researcher obtained the following data after administering attitude questionnaire to the students and the scores on academic achievement test. Using the data below calculate the Pearson r.

X	10	11	12	12	13	14	15	15	16	17	17	18	18
Y	5	8	9	4	7	6	8	9	10	10	12	14	13

Solution

Step:

(i) Find the mean for X and Y

(ii) Complete the composite table

(iii) = 80.90, = 87.25 = 107.72

S/N	X	Y	X_X -	Y_y -	xy	x^2	y^2
1	10	5	-4.5	-3.8	17.10	20.25	14.44
2	11	8	-3.5	-0.8	2.80	12.25	0.64
3	12	9	-2.5	0.2	-0.50	6.25	0.04
4	12	4	-2.5	-4.8	12.00	6.25	23.04
5	13	7	-1.5	-1.8	2.70	2.25	3.24
6	14	6	-0.5	-2.8	1.40	0.25	7.84
7	15	8	0.5	-0.8	0.40	0.25	0.64

8	15	9	0.5	0.2	0.10	0.25	0.04
9	16	10	1.5	1.2	1.80	2.25	1.44
10	17	10	2.5	1.2	3.00	6.25	1.44
11	17	12	2.5	3.2	8.00	6.25	10.24
12	18	14	3.5	5.2	18.20	12.25	27.04
13	18	13	3.5	4.2	14.70	12.25	17.64
	188	115			80.90	89.25	107.72
	14.5	8.8					

r = = = = 0.83

Calculating Pearson "r" using the Raw Score Method

The formula is given by r =

Let us use the same data in example above

Steps

i. Complete the composite table

ii. If N = 13, = 188, = 115, = 1744, = 2806 and = 1125 then

S/N	X	Y	XY	x^2	y^2
1	10	5	50	100	25
2	11	8	88	121	64
3	12	9	108	141	81
4	12	4	48	144	16
5	13	7	91	169	49
6	14	6	84	196	36
7	15	8	120	225	64
8	15	9	135	225	81
9	16	10	160	256	100
10	17	10	170	289	100
11	17	12	204	289	144
12	18	14	252	324	196
13	18	13	234	324	169
	118	115	1744	2806	1125

r = = = = 0.83

You can see that the two approaches give the same result. This is because the formula of the deviation is derivable the formula of mean score.

Now what is statistical decision on this result (0.83) of coefficient correlation?

r = 0.83 (is a high relationship)

18.10.2 Statistical Decision on Coefficient of Correlation

When correlation coefficient is test for significance at a $\alpha = 0.05$, the following decision may be taken:

(i) If 0.05 r 0.20 there is negligible relationship

(ii) If 0.21 r 0.40 there is low relationship

(iii) If 0.41 r 0.60 there is moderate relationship

(iv) If 0.61 r 0.80 there is substantial relationship

(v) If 0.81 r 1.00 there is high relationship

Spearman Rank Order Correlation Coefficient – rho

Thus was developed by Spearman and Brown and that is why it is sometime referred to as Spearmen-Brown Rank Order Correlation Coefficient.

The formula is given as rho = 1 -

Example

10 Students are ranked based on their achievement in Instructional Scaffolding and Traditional teaching in mathematics course. Calculate the coefficient of correlation between the two modes of teaching mathematics.

S/N	1	2	3	4	5	6	7	8	9	10
X	51	44	70	32	65	67	19	71	45	80
Y	49	41	45	31	50	61	11	64	21	75

Solution

Steps

(i) Complete the composite table by ranking the scores and difference between the ranks

(ii) Apply the formula: rho = 1 -

(iii) 1 - = 1 - = 1 - = 1 – 0.109 = 0.891

X - for Instructional scaffolding

Y - for Traditional Teaching

S/N	X	Y	Rx	Ry	D	D^2
1	51	49	6	5	1	1
2	44	41	8	7	1	1
3	70	45	3	6	-3	9
4	32	31	9	8	1	1
5	65	50	5	4	1	1
6	67	61	4	3	1	1

8	71	64	2	2	0	0
9	45	21	7	9	-2	4
10	80	75	1	1	0	0
Σ						10

Student Activity

1. Using any convenient correlation method, calculate the correlation coefficient of the data below:

S/N	1	2	3	4	5	6	7	8	9	10	11	12
X	31	24	50	12	45	47	09	51	25	60	15	10
Y	29	21	25	11	30	41	01	44	11	55	05	03

2. Explain what is meant by "coefficient of correlation". What does the sign (+ or -) tell you about the set of variables?

3. Two attributes or random sample of size 10, observed yielded the following data

X	15	17	18	25	19	32	21	27	28	30
Y	9	10	11	16	12	20	15	17	32	24

Compute:

(a) The Pearson Product Moment Correlation Coefficient r_{xy}

(b) The Spearman's rho

18.11 Inferential Techniques

Statistical tests are categorized into two groups, namely parametric and non-parametric tests. The parametric tests (parametric statistics) require that certain assumptions to be met in order for them to be valid. The followings are three very important assumptions that are made when using parametric statistics to test hypotheses.

1. The variable measured is normally distributed in the population
2. The normal distributions have the same standard deviation.
3. The data collected is from an interval or ratio scale

Therefore, the tests can be referred to as statistical procedures in which make inferences about the population parameters such as mean. In contrast, the non-parametric tests (parametric statistics) are statistical parameters in which inferences are not made about population parameters and no assumptions are made about them. Parametric test that are commonly used in mathematics education and educational research are the z-test, t-test and f-test. This work will only treat z-test and t-test.

18.12 Z-Test

The Z-statistic is used in testing hypothesis involving one sample i.e. determining whether two means are significantly different. It is usually adopted when the sample size is large i.e. when it is equal to or greater than 30. The formula for calculating Z-test is

$Z =$

Where $=$ The mean of a group

SD_x = Standard error of difference between means

$SD_x =$

$Z =$

Where $=$ mean of group I

$=$ mean of group II

SD_x = Standard error of difference between means

Example 7: Suppose a mathematics teacher has the following sets of scores in SSCE over the year.

X_1	3	4	5	6	7
X_2	2	3	3	3	4

Now, let us illustrate the use of Z-test in hypothesis testing with above example

X_1	$X_1 -$	$(X_1 -)^2$	X_2	$(X_2 -)$	$(X_1 -)^2$
3	-2	4	2	-1	1
4	-1	1	3	0	0
5	0	0	3	0	0
6	1	1	3	0	0
7	2	4	4	1	1

$= 25$ $= 10$ $= 15$ $= 2$

$= = 5$ $= = 3$

$SD^1 = = = 1.4$ $SD^2 = = = 0.82$

Now we have everything we need and all we have to do is substitute the correct number for each symbol

$Z = = = = = = 2.8$

Assuming we selected p = 0.05, what we need to do is to make reference to the table of Z-distribution. At 0.05 level of significance, the critical or table value of Z = 1.96. since the Z-value (2.8) is greater than the Z critical value, we reject the null hypothesis, otherwise we do not reject it.

18.13 The T-Test

The t-test otherwise called the student's t-test and is an inferential technique. It was developed by William Gosset in 1908. It is used to determine whether two means are significantly different when the sample size is small (that is n > 30). There are two different types of t-tests, the t-test for independent sample and the t-test for non-independent samples.

18.13.1 T-Test for Independent Samples

Independent samples are samples which are formed, that members of one group are not related to members of the other group but they are selected from the same population. The t-test of independent samples is used to test hypothesis whether there is probably a significant difference between the means of two independent samples. The formula is given as

t =

where = means of group I and group II

S^2 = Standard deviation

n = Number of subjects in each group

Example: Using the following sets of scores for two

A	Group	9	17	16	15	14	15	10	18	18	20	26	11	12
B	Group	6	10	12	18	13	1	11	9	19	5	15	10	

(a) Formulate a testable hypothesis

(b) Test the hypothesis at $\alpha = 0.05$

(c) Use the t-test for independent samples to test the stated hypothesis

Solution

First, let us label the score and then calculate the mean , then

Group A as X_1 and Group B be X_2

X_1	$X_1 -$	$(X_1 -)^2$	X_2	$(X_2 -)$	$(X_2 -)^2$
9	-6.5	42.25	6	-4.8	23.04
17	1.5	2.25	10	-0.8	0.64
16	0.5	0.25	12	1.2	1.44
15	-0.5	0.25	18	7.2	4.84
14	-1.5	2.25	13	2.2	96.04
15	-0.5	0.25	1	-9.8	0.04
10	-5.5	30.25	11	0.2	3.24

18	2.5	6.25	9	-1.8	67.24
18	2.5	6.25	19	8.2	33.64
20	4.5	20.45	5	-5.8	17.64
26	10.5	110.25	15	4.2	0.64
11	-4.5	20.25	10	-0.8	
12	-3.5	12.25			
201		= 238.95	129		= 248.44

= = 15.5 S^1 = = 18.38 S = = 4.29

= = 10.8 S^2 = = 20.70 S = = 4.55

Therefore t = = = = = 5.7

Since we selected α = 0.05, then we need to go the t-table with appropriate degrees of freedom. For the t-test for independent samples, the formula for the degree of freedom is P1 + P2 – 2. Using the formula above:

Df = 13 + 12 – 2.

tcal = 5.7

p = .05

df = 24

tcri. = 2.064

With these, since the t-calculated (t-value) required for rejection or upholding of the null hypothesis. And the t-value is greater than t-critical (2.064), we do retain the null hypothesis.

a. There is no significant difference between the mean scores of male and female students taught scaffolding learning strategy.

b. t-cal(5.7) >tcrit (2.064)

Decision: H_o is retain since the tcal>tcrit. The strategy is gender friendly.

18.13.2 T-test for Non-independent Samples

Non-independent samples are those that have undergone some sort of matching. The members of one group are systematically connected to the members of a different group when samples are not independent.In order to evaluate if there is likely a significant difference between the means of two matched or non-independent samples, or between the means for one sample from two separate times, the t-test for non-independent samples is utilized. The non-independent sample t-test formula:

t =

where d = difference between each matched samples

= Sum of the differences between the matched samples

D^2 = the square of the difference between each matched samples

N = number of degree of freedom

Example 9: Suppose a set of student in NCE II took test in both mathematics and statistics. Their results are as follows.

S/N	1	2	3	4	5	6	7
Mathematics	19	23	30	17	50	25	10
Statistics	29	63	46	37	58	80	43

Are the results significantly different?

Solution

X_1	X_2	D	D^2
19	29	-10	100
23	63	-40	1600
30	46	-16	256
17	37	-20	256
50	58	-8	64
25	80	-55	3025
10	43	-33	1089
		= -182	= 6534

Now substitute into t-formula.

t = = = = = = -3.97

Thus, t-value = -3.97 at α = 0.05, going t-table of non-independent samples with degree of freedom N – 1 = 7 – 1 = 6. Therefore, t-value = -3.97, p = 0.05 df = 6 and t-critical or table value of t = -2.365. since the calculated t-value of -3.97 is less than the critical value of -2.365 than H_o is reject.

Student Activity

1. What is Z-test? (b) Distinguish between Z-test and t-test
2. Suppose we have the following sets of post-tests scores for two groups

Group I: 9, 8, 6, 7, 2, 1, 4, 6, 4, 7, 8, 3, 6, 2, 6, 7, 3, 7, 8, 10, 9, 13, 16, 17, 6, 10

Group II: 8, 7, 5, 6, 3, 2, 3, 2, 3, 7, 3, 8, 7, 2, 2, 1, 3, 6, 2, 4, 0, 9, 9, 8, 12, 10, 25, 8, 5, 2, 5, 10, 25, 8

Find out whether these two sets of scores are significance different or not

1. Briefly discuss any four sampling techniques.
2. Describe any four tools used in data collection.
3. With appropriate research question, mention any four test statistics that can be used in data analysis.
4. Differentiate between: (a) Sample and population (b) t-test and z-test

5. For the topic "**The Study on Attitude and Academic Performance of In-Service Mathematics Teachers in Introduction to Probability**"

a. What are the instruments that will be used for data collection?

b. What statistical tool will be used for data analysis

18.14 Writing of Research Report

One of the important aspects of any research study is communication of the results to other researchers. This important aspect can be accomplished through a well written research report. Research reports can be of two types. These reports are in form a book called a project or thesis or dissertation and a journal. In any of the two, you must start with the title. The title is the label given to the report and must not be too long. After the title comes abstract, this contains a brief summary of the all study.

In the journal report or article (which unlike the project or thesis is not presented in pursuing a degree or other academic qualification), the format is different. Here you need introduction (which include research problem and review of literature), methodology including research design, sample, instrumentation and method of data collection and analysis. Other sections are results, a summary of discussion of findings with brief remark on how the results fit the previous studies, conclusion, recommendations and references.

The format for writing a report, thesis or dissertation varies from an institution to institution and from discipline to discipline. Research report is usual written in past tense. Essentially, a research project report comprises three major parts, viz:-

I. The preliminary section.
II. The main body.
III. The appendix.

The Preliminary Section

I. The title Page
II. Certification Page
III. Acknowledgement
IV. Table of Contents
V. List of Tables
VI. List of Figures
VII. List of Appendices
VIII. The Abstract

Chapter 1: Introduction

I. Background to the Study
II. Statement of the Problem
III. Purpose of the Study
IV. Research Questions and/or Hypotheses
V. Scope and Delimitation of the Study

VI. Definition of Operational Terms

Chapter 2: Review of Literature

I. Conceptual Framework
II. Theoretical Framework
III. Overview of Related Studies
IV. Implications of Literature Reviewed

Chapter 3: Research Methodology

I. Research Design
II. Area of Study
III. Population
IV. Sample and Sampling Procedure
V. Instrument for Data Collection
VI. Validation of the Instrument
VII. Reliability of the Instrument
VIII. Method of Data Analysis

Chapter 4: Data Presentation, Analysis and Discussion

I. Data Presentation
II. Summary of Findings
III. Discussion of Findings

Chapter 5: Summary, Conclusion and Recommendations

I. Summary
II. Conclusion
III. Implications of the Findings
IV. Recommendations arising from the study
V. Limitations of the Study
VI. Contributions to Knowledge
VII. Suggestions for Further Studies

Chapter 6: References

I. References
II. Appendix

References are the final state on reporting. Only the books and studies that the researcher referenced in his report are included as references. This does not apply to all of the sources he read but did not cite. However, if the researcher wishes to add a list of the books or papers, he has read but has not referenced, he is creating a bibliography rather than a reference.

Most projects, dissertations and theses report in educational research are usually organized in five chapters. Chapter one – the problem; chapter two – literature review; chapter three – research methodology; chapter four – data presentation and analysis; chapter five – summary, conclusion and recommendations.

18.15 Summary

In this chapter, you have the concept of statistics in research. Statistics involves collection, organization, representation, analysis of data, interpretation of data and decision making. You also learnt the types of statistics and benefits of statistical analyses in educational research.

In this chapter sequencing order is discuss as arranging of scores either ascending or descending order and also rounding of numbers to required figures. You learnt about graphical representation of data and finding measure of tendency. Lastly simple percentage and inferential statistical tools are discussed.

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CHAPTER NINETEEN

EVALUATION IN MATHEMATICS EDUCATION

19.1 Objectives

By the end of this chapter, you should be able to:

i. Define Measurement and Evaluation
ii. State the types of Measurement and Evaluation
iii. To distinguish between Measurement and Evaluation.
iv. Purpose of Measurement and Evaluation

19.2 Introduction

You are introduced to certain key ideas in this chapter that are connected to determining whether or not goals have been attained. The chapter essentially walks you through what measuring and assessment entail in math instruction. Additionally, their kinds are covered. You should be aware of the subtle differences between these ideas and their respective purposes.

19.3 Measurement

Measuring is the act of assigning numbers to things, quantities, or events in accordance with a well-defined rule or standard in order to ascertain their value, height, and volume. In other words, measuring is the systematic method of determining the presence of a property. Physical measurement and psychological measurement are the two forms of measurement. Sometimes, physical measurement is referred to as empirical measurement. It refers to the use of measuring instruments, such as tapes, rulers, thermometers, clocks, speedometers, etc., to estimate the size of anything, particularly in reference to a recognized standard. While psychological measuring (educational measurement) entails the use of educational instruments such as tests, interviews, examinations, and observations, this is done to assess intangible characteristics such as intelligence, aptitude, and other quantitative factors.

19.4 Measurement Levels

There are several degrees of measurement based on what is to be measured, the equipment to be used, the characteristic being measured, and the desired level of precision. There are four scales of measurement: nominal scale, ordinal scale, interval scale, and ratio scale.

Nominal Scale: This scale is designed for identifying purposes only and only allows classification into classes or categories. Measurements based on this scale include classifying individuals by gender (male, female), tribe (Nupe, Gwari, Kambari), religion (Muslim, Christianity), and marital status (single, married). Their categories have no associated magnitude, and these scales reflect the lowest level of measurement. For example, the course codes MAT124 and MAT225 cannot be combined or subtracted to make sense when using this scale of measurement.

Ordinal Scale: measuring on this scale permits classifying and ranking and bears the "order" quality. On this scale, the assignment of rankings (or places) to students based on their test scores is an example of measuring. Other variables employed on this scale include merit, reward, rating, etc. This scale allows for the correlation or comparison of variables. In other words, the student ranked first performed better than the student ranked second. However, no mathematical operations other than comparison are possible in this scale. You cannot say position 3rd plus 2nd or position 3rd minus 2nd, nor are the gaps between distinct ranks or positions always equal.

• **Interval:** this scale is classifiable, uniformly spaced, and possesses the quality of order. Examples of measurements on this scale are examination scores or grades. The difference in performance between scores 40 and 45 will be equivalent to the difference in performance between scores 10 and 15. Test (examination) scores, the degree of temperature, and all psychological measurements employ interval scales of measurement as independent variables. This scale is used for all mathematical operations except division, as interval scale has no zero value. For instance, if a student receives a score of 0% on an exam, this cannot be construed as a sign of ignorance or lack of intelligence; 0% is not absolute. In other words, it does not correlate to a lack of the attribute in its entirety.

Ratio Scale: this scale possesses all the characteristics of the interval scale, including the ability to be classified, ranked, uniformly spaced, and the presence of a natural zero. On this scale, 0 represents the lack of the attribute being assessed. For example, if the length of an item is 0 centimeters, that thing does not exist. Time, weight, height, distance, etc., are variables that employ ratio scales of measurement. This scale permits all mathematical operations, and the physical sciences employ this degree of measurement. The ratio scale is the most polished, exact, or accurate of the scales, followed by the interval scale and the ordinal scale in that order. The nominal scale has the lowest degree of precision. The first three levels of measurement in mathematical education are nominal, ordinal, and interval.

Attempt 1

1. What exactly is measurement?
2. Mention the two fundamental types of measurement.
3. What are the four measuring scale levels?

4. Which of these has the only ordering property?
5. Describe the characteristics of the interval and ratio scales
6. Provide two measurement examples for each of the four scales.

19.5 Measurement Inaccuracies

The definition of measurement error is the difference between the real or actual value and the measured value. The inaccuracy may originate from several sources and is often characterized as follows:

These varieties are:

1. Gross Errors

Systematic Errors 2.

3. Random Errors

19.5.1 Gross Errors

This error is caused by human error, such as when the individual operating the instrument takes the erroneous reading or records inaccurate data. This blunder falls under the category of human error and is characterized as a big error. The major mistake can only be prevented by attentive reading.

19.5.2 Synchronous Errors

These mistakes are inherent to instruments due to their mechanical design. They may be the result of production calibration or device usage. These inaccuracies may result in an erroneous reading that is too low or too high. Additionally, the issue might be caused by operator error. Good equipment utilized in an uneducated manner provide a tremendous outcome. For instance, improper usage of the instrument may result in the inability to modify the zero of the instrument, inadequate initial adjustment, and excessive resistance. These faulty procedures may not cause irreparable harm to the instrument yet may nonetheless result in mistakes.

19.5.3 Random Errors

This sort of inaccuracy is referred to as random error, and it is generated by rapid changes in the meteorological conditions. These forms of mistake exist after systematic error has been eliminated. Hence such type of error is also called residual error.

19.6 Evaluation

Evaluation may be considered an inherent component of every educational activity plan, regardless of the duration of the teaching. The evaluation procedure is ongoing

throughout the duration of education. It is also regarded as the last action that concludes the teaching-learning process, regardless of whether the instruction lasted a year, a semester, a teaching unit, or a single period. The analysis of these definitions reveals that assessment as an important component of the teaching-learning process consists of three parts. Which are.

1. Identifying and describing the desired results
2. Creating or selecting tests and other assessment instruments related to the targeted aims.
3. Using the evaluation data to improve learning and teaching.

When discussing evaluation in the mathematics classroom, we attempt to determine the quantity and quality of students' mathematical comprehension and success based on clearly specified goals. However, emphasis is placed on making instructional objectives explicit and quantifiable.

19.7 Types of Assessment

There are four forms of evaluation: placement, formative, diagnostic, and summative.

19.7.1 Placement Evaluation

This is the evaluation performed in order to categorize pupils into the right category. Before allocating students to the sciences, arts, commerce, and technical tracks, several schools provide tests to determine their performance. This form of evaluation is conducted by the teacher to determine the entrance behavior of each pupil.

19.7.2 Formative Assessment

Formative assessment is an evaluation aimed to aid both the teacher and student in identifying parts where the student failed to learn. It offers feedback to the instructor and student to help them address problem areas. This is accomplished by weekly tests, final exams, etc. As an effective mathematics instructor, you must always assign homework or classwork after each lesson in order to receive feedback on your teaching.

19.7.3 Diagnostic Assessment

This kind of assessment is meant to be used after formative evaluation. You employed formative assessment as a math teacher to pinpoint your pupils' areas of weakness, and you subsequently used remedial strategies to address recurrent problems. You will now create a type of diagnostic exam that is used in classroom education to ascertain the underlying reason why pupils continue to have learning challenges. These diagnostic tests might be in the form of a performance test, a self-rating, an interview, an observation, etc.

19.7.4 Summative Assessment

This is the form of assessment that is created and conducted at the conclusion of a period of instruction or a course of study. It is termed a summary because its purpose is to establish the extent to which the educational aim has been met. It is also designed to assign students course grades and certificates. The awards for educational certificates, such as NCE. Diplomas, degrees, WAEC, and NECO, among others, are awarded through a final test, which is an example of summative assessment. This will determine the degree to which the program's broad objectives have been met. It focuses on the objectives, development, and results of the teaching-learning process.

19.8 Functions of Assessment

The purpose of evaluation in mathematics education may be categorized into four major categories, which are as follows.

i. Instructional capabilities

ii. Administrative functions, iii. Counseling functions, and iv. Research functions.

19.8.1 Educative Functions

These functions pertain to the ways in which evaluations assist in enhancing the quality of classroom activities. Instructional functions are comprised of all evaluation functions that can lead to enhanced teaching and learning in the classroom and result in highly effective instruction. The particular instructional roles of educational evaluation include.

i. Good study habits: regular evaluation or testing causes pupils to often update their schoolwork. And this promotes healthy study habits in the student's thinking.

ii. Motivation: evaluation enhances students' motivation anytime they get feedback on performance that motivates them to work more, regardless of whether the input is good or negative. If the evaluation is favorable, the kids are happy and will work diligently to maintain their performance. If the grade is unfavorable, the student will be motivated to improve their performance.

Evaluation offers information to the instructor regarding the students' entering habits. Any effective classroom instruction must document students' prior knowledge or lack thereof at the outset of the course.

iii. Determining the extent to which objectives have been realized: Evaluation helps to offer information that assists the instructor in determining the extent to which the educational objectives have been attained. At the conclusion of the lesson, information indicating the extent to which the instructional objectives have been met is made available.

feedback on instructors' and students' areas of weakness and strength

Evaluation gives teachers and students with information on their weaknesses and strengths. Students will determine where their strengths and weaknesses lie. On the other hand, the instructor will see which educational strategies are effective and which are not.

19.8.2 Administrative Capabilities

Evaluations have the following particular administrative purposes in mathematical education and education in general:

Students are classified based on their talents and interests using evaluation data. This information helps us determine what sort of course a student should take, whether he or she should enroll in teacher education, secondary education, or technical education.

Placement of students: the information offered by evaluation aids in placing students at a point or grade in a course that is most suited to their individual qualities. It also involves placing the student in the most suitable career. An example of placement would be placing a gifted kid in a scientific class rather than an arts class.

Selection: a specialized administrative function of evaluation that aids in selecting individuals and curricular materials that fulfill the institution's best interests. The school chooses whether a new set of curricular materials (textbooks, equipment, and supplies) will be selected to promote the institution's goal accomplishment based on assessment comments.

Certification: the data collected during assessment served as the foundation for providing certificates to students who successfully completed a program.

19.8.3 Guidance Functions

Evaluation in mathematics and education fulfills the following particular guiding roles:

i. Diagnosis of student learning challenges: the information produced by evaluation aids in the diagnosis of students learning difficulties. This information is to guarantee the right educational growth of the student through programme of remediation.

ii. Solving societal and individual issues. The evaluation results will help the school counselor to assist kids in resolving their social and personal issues. Numerous kids have multiple social and personal issues, such as family issues, peer group conflicts, and adjustment issues, among others. The school counselor will be able to support the student in coping with this sickness if the evaluation yields appropriate information.

iii. Occupational career: an additional specialized counseling role offered by educational evaluation is supporting the student in making an appropriate and effective vocational selection. It is vitally important to find a career that matches one's aptitude, interests, and skills.

19.8.4 Research Capabilities

Evaluation in mathematics and education serves several important research purposes, including:

i. Effectiveness of educational techniques requires study to determine which instructional approach is more successful. Evaluation gives the evidence required for appropriate conclusion formation.

ii. Effectiveness of instructional materials: Research undertaken in the domain of instructional material effectiveness will aid in determining which materials will be effective in achieving certain educational objectives in mathematics. This research is made possible through evaluation.

iii. New curriculum effectiveness: research assistance in determining the efficacy of a new curriculum prior to its implementation in a school system. The evaluation information aids in generating judgments on the efficacy of the new curriculum.

iv. characteristics and influencing elements of the learners Through research, we have been able to comprehend the characteristics of the learner as well as other factors that influence learning.

Activity

1. Provide an explanation of educational measurement and evaluation
2. Acknowledge measurement errors
3. Mention the four levels of measurement.
4. Explain the meaning of summative evaluation
5. Give two examples of the kind of evaluation
6. State the four major functions of evaluation in mathematics education
7. What specific guidance functions does evaluation serve in mathematics education
8. State the specific research function which evaluation serves in mathematics education

19.9 Summary

In this chapter, the following points shave been highlighted.

§ Measurement is the systematic process for finding the extent to which an characteristic is present in an object in numerical terms; evaluation has to do with making valid judgment or decision based on the information obtained through evaluation.

§ Measurement is of two kinds, physical and psychological. Physical is carryout by the use of measurement instruments, such as clock, thermometer, calendar, etc. Educational measurement is carryout through test, interview, etc.

§ Errors in measurement and its classification gross, systematic, random were presented and discussed.

§ We spoke about four different measuring scales. These are the ratio scale, which contains absolute zero and is particularly utilized in the physical sciences, the ordinal scale, which uses rank order, the interval scale, which uses equal intervals and equal amounts but without absolute zero, and the nominal scale, which involves easy identification.

§ Evaluation can be classified into placement, formative diagnosis and summative

§ Evaluation in education serves four major functions-instructional, administrative, guidance and research functions.

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CHAPTER TWENTY

CLASSROOM TEST

20.1 Objectives

By the end of this chapter, you should be able to:

i. define classroom test
ii. list the different types of tests
iii. state the purpose of classroom test
iv. list some pitfalls in classroom test
v. list some pitfalls in teachers – made test
vi. Enumerate the characteristic of good test

20.2 Introduction

The term "classroom test" also refers to tests created by teachers. A test is crucial to classroom teaching and learning because it indicates how well pupils have learned. The exam in the classroom might be a performance or accomplishment test, as well as any other kind of test, such a practical examination, created by the subject teacher for particular purposes depending on what was taught over the course of a certain amount of time.

This chapter will discuss the sorts of tests, the characteristics of good tests, their aims, and other facets of classroom assessments.

20.3 Classroom Test

The term "teacher-made exam" is commonly used to describe the exam given in class. It's a device or instrument for measuring and assessing. It is a key educational tool used to assess students' learning outcomes. In this view, a test is just a set of questions that students are required to respond to, and their answers serve as a measure of their performance. It is not a certainty that exams given in class would include well-written questions. For an unskilled educator, coming up with good questions or materials is not always simple. There are a few guidelines that must be followed while creating successful questions or items in order to guarantee that the results provided for the tests will be consistent and useful. Examining the function of classroom examinations.

20.4 Objective of Classroom Examination

Testing in the classroom serves the apparent purpose of assessing what pupils have learnt following the conclusion of a course or unit. Some of the justifications for testing are as follows:

i. Tests are undertaken to find out if the objectives given for a particular course, lesson or subject is reached or not.

ii. Students are assessed in the classroom to gauge their individual development over a given time period. This allowed the instructor to determine whether or not the pupils were advancing in the course, topic, or lesson. If progress is not achieved or is sluggish, the instructor intensifies instruction to accomplish it.

iii. Tests are used to assess whether or not pupils have learned in class. Exams indicate whether or not students have mastered a certain course or topic. Thus, the instructor can re-teach for enhanced learning.

iv. Students are placed into a certain class, school, level, or work position depending on the results of a placement exam.

Tests highlight the difficulties or areas of difficulty for students. Thus, the instructor employed exams to diagnose or identify the pupils' difficulties or areas of difficulty.

The purpose of tests is to anticipate results. The instructor uses tests to forecast whether a student will be able to complete a specific activity or perform well in a specific school, college, or university.

20.5 Examination Pitfalls in the Classroom

The following are observations made concerning classroom tests (tests created by teachers) that you should avoid while creating test questions for your class:

i. Exams, in the opinion of the majority of educators, do not align with the different levels of learning outcomes. The teachers describe their learning goals, which span from simple memory through evaluation. The teachers' materials, however, are limited to one level.

ii. Many of the examination exercises do not measure what they are intended to assess. In other words, the majority of classroom assessments are invalid.

iii. Some teacher-made exams do not thoroughly cover the material given. A quality of a good exam is that it should cover all of the material presented.

Most lecturers lack clarifications in the phrasing of their prepared examinations. Occasionally, the exam items are vague, imprecise, unclear, and poorly phrased.

The majority of classroom assessments fail the analytical exam. They lack sufficient discrimination and are not constructed according to difficulty levels.

20.6 Types of Classroom Examination

There are several sorts of examination forms used in classrooms. This can be a discrete, integrative, norm-referenced, criterion-referenced, essay, or objective examination. All of these are tested often in class. Discrete point exams are supposed to evaluate a single item or ability at a time, whereas integrative tests incorporate several things, structures, and skills.

A norm-referenced exam is a standardized test meant to rate pupils from high to poor on the basis of test performance. Such tests include JAMB, GCE, SSCE, and teacher-made exams, among others. In contrast, a criterion-reference exam is a test that measures student success in relation to defined objectives.

However, this course will focus on the essay and objectives tests. These are the typical tests that you may simply create for your own purposes in class.

20.6.1 Essay Test

The colonial education system brought this form of examination, which is a free-response exam, to this country. In this circumstance, the Testee or the pupils are responsible for formulating answers to questions posed. In addition, they are permitted to express or articulate the responses in their own words. There are two types of essay tests: extended response and restricted response (short – essay)

Extended response or Free responses

20.6.2 Extended Responses or Free Response Essay Test

In this essay examination, the information required from Testees or students is not constrained in any manner. Students are granted total autonomy in planning and organizing their responses. A student communicates their thoughts by freely, accurately, and clearly expressing themselves in their own words. Now let us offer some instances.

i. Provide an explanation of measuring in mathematics education
ii. Describe mathematics education evaluation

20.6.3 Response Type Restricted

In this sort of question, the Test-Taker or student's answer options are restricted to those presented. The supplied responses are quite limited. Sometimes referred to as the short essay test.

Example 1: Provide two benefits of essay testing.

Example 2: List four level of measuring scales

20.6.4 Advantages of the Essay Test

The essay test is a time-honored, traditional kind of written examination that provides a number of advantages in mathematics measurement and assessment education. Some of these benefits include:

i. It is simpler to create than an objective exam.
ii. It offers pupils the chance to practice and improve their writing abilities expressiveness.

iii. It decreases the likelihood of copying or other forms of examination fraud.

iv. It permits the examiner to seek the student's viewpoint without prejudging it, as is the case with the selected-response item.

It is a potent tool of self-expression and thought organizing.

20.6.5 Disadvantage of the Essay Test

1. Essay test grading requires more time than objective test grading.
2. Scoring is subjective since it is not dependable, as different examiners might assign different scores to the same response.
3. Because teachers often provide a restricted number of questions for essay tests, course material is typically not addressed in full.
4. It is often difficult and time-consuming to grade essay questions, particularly in big class situations.
5. Individual pupils with a strong command of language had an advantage over those with a weak command of language, who were at a disadvantage.

20.7 Test of Objectives

The objective test is also a teacher-created test in which the items and questions are carefully crafted to restrict the types of responses students can submit in order to pass the examination. For a student to receive the proper response, he or she must exhibit the appropriate knowledge, comprehension, and abilities.

There are two types of objective test items: those that ask the test taker or student to offer the response, often known as supply test items, and those that are either short answer or full type tests. The terms arrangement, true/false, matching, and multiple-choice refer to those that ask the Testee or student to choose from a list of possibilities. The specifics of the categories for objective tests were shown in the image below.

20.7.1 Supply Test Items

This kind of test item requires the test taker to provide very succinct responses to the questions. These solutions might consist of a word, phrase, number, symbol, etc. This test's creator must be very careful with language and ensure that each question has just one possible response. Provide test items in the form of.

a. Test questions with a brief answer (A): These questions requested the test taker to give a succinct response. It is a straightforward inquiry that may be used to assess elementary school students' simple learning outcomes in lower and upper basic classes.

Examples of short answer items are:

1. What is the formula for area of a square
2. The sum of angles of a parallelogram is

b. The testee is required to furnish or fill one or more missing words in a phrase. The completion questions can need a single response for a blank or a sequence of replies for many blanks. It may be used to assess factual information, particularly in elementary schools where students are only learning how to communicate with others.

1. The line drawn from centre of a circle to the circumference is called
2. Square has ……..side and ……..angles
3. ……..is a special chord passing through the of circle

20.7.2 Selection Test Items

This is the type where possible options are provided for the Testee to choose the correct option. Let us discuss sub-division of selection test items one after the other.

a. The Testee is asked to indicate whether a statement is true or false, yes or no, agree or disagree, etc. in this form of true-false question. examples: Choose true or false for the following statement.

i. A square has four sides equal
ii. 45^0 is not half of right angle
iii. A right-angled triangle is one with equal sides and angles.

b. Multiple – choice: in this type, the item consists of a problem and suggested solutions from which the Testee selects a response. The problem may be direct question or incomplete statement called stem, while the listed responses are known as alternatives. The correct alternative is called answer. While the other alternative are referred to as distracters because they are deliberately constructed wrong answers.

Example of multiple choice items are:

1. The following is not a graphical method of representing data

a. Pictorial b. Bar chart c. Mean d. Histogram

2. A type of average that shows the most popular items is

a. Mean b. Bar chart c. Variance d. Mode

3. Ability to communicate data concisely and intelligently depends on the knowledge of:

a. Measurement b. Formular c. Rules d. Statistics

20.7.3 Matching Items

During the matching test, there are two concurrent columns of questions. Words, numbers, symbols, and phrases are shown in one column, and they must be matched with a word, phrase, or other potential response from the list in the other column. Premises refers to the items in the column that look for a match, whereas answers refer to the things in the column that are used to make the decision.

Matching items are useful in measuring the testee's ability to associate, relate or interpret elements or events learnt in the classroom. And it is good for all classes in the primary schools

Example: in each blank space in column A write the correct option in column B

Column A	column B
1. The child is sleeping at	morning
2. The cock is crowing early in the	evening
3. The two men are playing AYO" game in the	N16:25K
4. N5 + N5 + N6 + N25k	N37:25K
5. 25K + N25 + N5 + 7	night

20.7.4 Advantages of Objective test

The common usefulness of objective test items is as follows:

1. The test components are scored objectively in comparison to the essay test that is being taken.
2. It is effective for gauging factual knowledge.
3. It demonstrated that there was opportunity for pre-test through item analysis that was standardized and reused several times if handled appropriately.
4. Objective exam scoring is simple and takes minimal time.
5. Compared to an essay test, the objective test gives the teacher the chance to cover more material.
6. The test items are a more accurate and dependable indicator of the performance of the test taker.
7. The objective-type exam questions discourage memorization and promote critical thinking.

20.7.5 Disadvantages of Objective test

The weakness of the objective test items results largely from the fact that the responses are provided for the Testee. Specifically, the disadvantages associated with objective test

items are discussed below.

1. In the objective type, the test items don't put stress on the ability of organization of subject matter learnt by Testee
2. The construction of a good objective test is time consuming and difficult
3. Objective tests fail to check cramming
4. This type of test the items are not used for the purpose of diagnosing the learning difficulties of the students
5. In objective test, students don't get scope for making comparisons
6. The objective test type doesn't give Testee opportunity to express opinion, neither given the chance for creative thinking.

20.7.6 Characteristics of Good Test

For a mathematics teacher to have good test, it must not be done careless or haphazard. A qualities mathematics teacher must observe some qualities, whether the test is diagnostics or achievement test. A good test has the following characteristics features:

1. Practicality: A successful exam should be simple, engaging, and convenient for both the teacher and the student. Therefore, there is no trouble.
2. Objectivity: A good exam must accurately reflect the goals of teaching and learning. The exam should be aware of the learning and testing objectives. For instance, testing should focus on that if the learning purpose is to master a skill and apply it.
3. Comprehensiveness: A good test should be thorough, covering all of the subjects that are being examined.
4. Validity: A reliable test must be reliable. If the measurements made are those that should be made.
5. Reliability: A trustworthy test must be one. This entails regularly measuring what it claims to measure. On a dependable test, the tester is sure that a subject will receive roughly the same result whether taking it on several times or by a different examiner.
6. Carefully and methodically chosen: For a decent exam, the materials must be carefully and methodically chosen so that they are of varying degrees of difficulty (not too easy or too difficult)
7. Discrete point and integrative: For a more thorough depiction of teaching-learning points, a good test should integrate discrete and integrative test processes. Both the discrete and integrative components of the topic matter should be included in the exam. A good test should incorporate all different learner's needs, range of teaching-learning situations, objective and subjective items.

20.8 Summary

In this chapter, you have studied

- § The teacher made tests and its types, used in the classroom.
- § The purpose of test, the advantage and disadvantage of essay and objectives test
- § Also, the characteristics of good test

Student Activity

1. State three advantage of objective test
2. State three advantage of essay test in school
3. Differentials between norm referenced and criterion reference test
4. State the two basic essay types of tests
5. Explain restricted and non-restricted essay types of tests
6. Explain the following word

a. Premises in matching words
b. Discrete point test
c. Integrative test
d. Testee
e. Responses in matching items.

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CHAPTER TWENTY-ONE

TEST DEVELOPMENT

21.1 Objectives

By the end of this section, you will be able to:

i. recognize the steps involved in creating a test for a classroom setting,
ii. prepare a table of specification
iii. carry out moderation process of test item
iv. carry out test item writing

21.2 Introduction

Testing essentially consists of four main steps: creating and developing the exam, administering the test, scoring the test, and reviewing the test results. Each of these actions can lead to a legitimate assessment of every Testee as well as a trustworthy test. You will discover how to construct tests for use in the classroom in this chapter. Additionally, construct the specification table. Following that, you will learn how to create the specification table, control the output of the item, and prepare the item for usage.

21.3 Test Development

The primary purpose of teacher-created assessments is to acquire valid, trustworthy, and valuable information about each student's performance within a certain period. This requires good preparation such that the sampling of exam questions is representative of the entire cog native domain. A good examination designed to measure particular learning objectives should assess fundamental course material. If mathematics learning outcomes are to be assessed, the test should be based on what pupils have studied in mathematics, not in physics or geography. The test is best seen as a sample if the questions are drawn from the extensive material provided. Priority number one when picking a sample test is creating a high degree of validity. This entails the creation of a test that includes the material taught, as well as the weighting of each item so that each topic studied is emphasized proportionally to all other topics, in line with its relative significance. Special care must be taken to prevent having too many or none on a particular issue, or too few or none on others. An description of the sampling technique for the classroom exam is provided below.

21.3.1 Steps in Creating a Classroom Examination

Tests administered in the classroom are an assessment tool used to examine the level of achievement made by instructor and student during classroom instruction. Numerous professors create tests, which frequently fail owing to inadequate and poor design; therefore, tests must be well-planned to fulfill their goal. The following serves as a guidance for designing a classroom examination.

§ Establish the purpose of the test.

§ Clearly state the learning objectives and assessment criteria; Establish the relative weight of each topic;

§ Choose the exam type (essay or objective) that is most suited;

§ Create a test blueprint to direct the design of the test.

§ Create test questions that are relevant to the learning objectives listed in the test plan.

§ Establish the scoring system and the findings' interpretation.

§ Establish the duration and length of the exam;

§ Put the components together into a test, give instructions, then administer the test.

21.3.2 Identify the Instructional Goals

The instructional aim is also known as the learning objective or course objective. It is a goal that is anticipated to be reached throughout the term of classroom education. When constructing test items for any course, the learning objectives of the course must be carefully examined. Because it is a desired learning result, students are expected to possess it upon completion of the course. The assessment of behavior in the cognitive domain of the education goal contains six level objectives: knowledge, understanding, application analysis, synthesis, and evaluation. The instructional objectives for this objective are specified. The instructional objectives and course material serve as the foundation for test creation.

21.3.3 Item Protection

The test developer should outline all of the material that will be assessed. This weighting is determined by the significance and attention placed on the content section. Item coverage is required since it is the way by which objectives are to be attained and mastery assessed.

21.3.4 Specification table/Test blueprint

Sometimes referred to as the educational blueprint. The educational blueprint is a two-dimensional chart that ties the subject matter subjects to the instructional aim in terms of behaviors and gives information regarding the focus to be placed on each sort of learning come. Table of Specifications is the shorthand name for a plan prepared by test developers (Test blue print). To create a test blueprint, you require a list of covered subjects and expected actions. The horizontal dimension of a subject-specific chart indicates cognitive domain activities, whereas the vertical dimension represents a list of subject-specific topic research. For testing, action verbs such as identify, sum, solve, simplify, evaluate, and explain are employed for the behaviors dimension, which typically consists of six objective levels (knowledge, compression, application, analysis, synthesis & evaluation.) Consider the following instructional aim if, for example, a mathematics instructor is tasked with creating an end-of-semester examination in mathematics.

§ Students should be able to do fundamental decimal operations.

§ Students should be able to determine the provided number of significant and decimal places.

§ Students must be capable of factorizing and solving quadratic expressions.

§ Students should be able to solve two-variable inequalities.

§ Students should be able to solve issues using all fundamental triangle kinds.

§ Students must be able to compute the mean, median, mode, and median.

On the basis of these aims, the relationship between the intended themes and behavior should be examined. The class attention on the subjects and the number of students' activities determine the relative value of simple topics and behavior outcomes.

The following are topic-specific information selected by the instructor.

1. Subject: Number and Numeration
2. Algebra Processes 30%
3. Geography 20%
4. Everyday figures 10%

As percentage weights are allocated to each content, as necessary. If 20 things are to be created, the following will determine the number of items to be specified for each topic:

§ Quantity and Numeration 40% of 20 things equals 8

§ Algebra Processes 30% of 20 things is 6 items

§ Geometry 20% of 20 things equals 4 items

§ Normal Statistics 10% of 20% equals 2 things

The most appropriate method for weighing the material of an objective exam is to allocate the number of test items to be completed for each topic. More questions are given to the more significant while fewer are given to the less significant. Thus, the

overall number of test items is related to their weight in the entire exam (that is, to educational objectives at higher and lower cognitive). In our example mathematics examination, forty percent of the questions should focus on number and numeration (8items) 30% of the exam questions for Algebra processes (6-items), 20% of the test questions for geometry (4-items), and 100% of the test questions for daily statistics concern algebraic processes (2items). As illustrated in the table below, the table of specifications for an essay test is created by designing a two-dimensional chart with contents vertically and objectives horizontally:

TABLE OF SPECIFICATION/TEST BLUEPRINT

CONTENT	Knowledge	Comprehension	Application	Analysis	Synthesis
Number and Numeration	2	--	2	2	1
Algebra Processes	2	1	1	2	--
Geometry	1	1	1	1	--
Everyday Statistics	1	1	---	----	---
Total	6	3	4	5	1

Once a thoroughly crafted table of specifications has been created, the test developer will have the direction necessary to construct test questions with a high degree of content validity. It should be emphasized that while choosing the exam style, both objective and essay questions should be employed appropriately depending on the course objectives and the desired behavior outcomes.

21.4 Item Format

Using the test blueprint, the professional test constructor is then responsible for executing the actual test items as designed. Following are rules for writing items:

i. As a test developer, the table of specifications is the blueprint for your item authoring and should be adhered to.

ii. The test developer should produce a larger number of items than specified in the test blueprint. This will allow for items that do not pass item analysis hurdles.

iii. In item writing, the test creator should be clear and avoid misleading terminology.

iv. In addition to generating test items at the proper level of difficulty as defined in the test blueprint, the test developer must also employ the relevant action verbs for each cognitive domain level. Included are the terms describe, explain, debate, enumerate, create, draw, compare and contrast, etc.

v. a suitable scoring guide must be developed at the time of item creation.

vi. To verify test items, subject specialists/experts should be included, and their observations should be taken into account while drafting the final item.

vii. Reviewing and picking the best things based on the test blueprint should follow the writing.

The instructions for all tests must be clear, precise, unambiguous, and goal-oriented in order to suit the learning goals and requirements of testers.

21.5 Test Moderation

Examining and critiquing test exercises by one or more specialists is a component of test creation. This procedure is known as test item moderation. After the construction of exam items, they are vetted by a school's subject head or a topic specialist who provides insightful feedback. After any required revisions and modifications, the test items are sent to external assessors (subject experts or specialists) for their comments. This must be sent together with the marking system for carrying out successful exercises in order for the exam to have construct and content validity.

21.6 Summary

In conclusion of this chapter:

§ You have acquired knowledge of test development as a strategy to direct the building of test items.

§ The Table of Specifications, also known as the Test Blueprint, is the building blueprint for the test.

§ The test blueprint instructs the test builder on how to develop a test. This provides construct and content validity to the test.

§ Test item authoring guidelines were outlined and discussed

§ The internal and external moderation of examination items was considered.

Student Exercise

1. Mention significant construction steps

2. What is a blueprint for a test?

3. Describe three functions of the test blueprint.

4. Provide a summary of the test blueprint.

5. A mathematics instructor devoted twenty weeks to the following topics:

Topic	**Number of weeks**
Number & Numeration	4 weeks
Areas of Plane shapes	5 weeks
Algebra Processes	1 week

Bearing	3 weeks
Latitude & Longitude	2 weeks
Everyday Statistics	5 weeks
Total20 weeks	

Using the information in the above, construct a table of specification for a 60 items test that will ensure adequate representation of the topics covered within this period.

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CHAPTER TWENTY-TWO

CLASSROOM ADMINISTRATION AND SCORING

22.1 Objectives

By the end of this chapter, you should be able to:

i. know test administration and its procedures
ii. define marking scheme and its preparations
iii. explain procedures for scoring
iv. convert scores to grades
v. carryout item analysis

22.2 Introduction

As you will recall, we had discussed the test construction as one of basic step in testing. In pursuance of reliable and valid judgments of every student that participant in a classroom test, they need to discuss test administration and techniques of scoring and grading. The purpose of this chapter is to explain test administration and different techniques of scoring and grading scores of students.

22.3 Test Administration

Refers to the array of actions designed for an examination program to decrease measurement error and improve the chance of a fair, valid, and accurate evaluation. Specifically, proper standardized processes increase measurement by enhancing test reliability and consistency. Exam administration processes are designed to enhance uniformity, assure test security, and protect the fairness and reliability of examination outcomes. These processes are followed before to, during, and following testing. Prior to testing, the test manual should be created to cover all regulations governing the administration of the examination. In addition, training should be provided so that all test administrators are aware of their duties in the administration of tests. Before delivering a test, it is also appropriate to identify and assign the required testing rooms. All exam materials should be adequately protected.

During testing, test administrators should be present in the testing environment (hall, etc.) in order to give technical support for any issues that may emerge. During operational testing, it is also the responsibility of the test administrators to ensure that all test security rules and procedures are adhered to. All testing irregularities (such as student misbehavior, copying the answers of others, accessing banned materials for aid, and the use of electronic devices, etc.) that may occur during testing must be disclosed and regulations must be followed throughout the duration.

After testing is complete, all secure and unsecured testing materials must be returned to a secure location for reconciliation. Before leaving school for the day, the test administrator should pack and secure test materials in the safe storage place. All the processes before, during, and after fluid test administration should be considered and followed to ensure uniformity and test security.

22.4 Scoring and Evaluation of Classroom Examination

As a course lecturer or topic instructor, one of your primary responsibilities is to score (or mark) and accurately evaluate student responses. This is the case since the final evaluation of the pupils depends on their scores (or grades). On occasion, incorrect scoring and grading systems have resulted in incorrect student evaluations. The objective of this section is to teach several approaches for scoring and grading student responses.

22.4.1 What is a scoring system?

It is a system that awards points for accurate answers or performance in an examination or competition. It is a strategy or set of standards used to evaluate the written output of students. This allows students to access examination papers to determine how the grading method was used.

22.4.2 Marking Scheme Needs

Designing and employing a marking scheme has a variety of advantages for both the instructor and the student. This includes

i. All students who took the same exam are treated equally since the marking scheme offers a standard basis for measurement.

ii. Since all students are treated similarly, the marking scheme enables a more objective approach of grading them.

iii. If the course lecturer or subject instructor who created the exam is unavailable, another expert can utilize the marking system to grade the pupils.

iv. a decent marking system can serve as a basis for question preparation if it is created in advance of the exam.

The adoption of a marking scheme enables students to self-evaluate and also facilitates peer evaluation.

The use of a marking system explains exactly how a student's grade is determined and how each mark is accounted for.

22.4.3 Designing Marking Scheme

The following steps can be taken to create a grading scheme for either an essay or an objective type test:

§ Write a sample response to each question, if the topic allowed.

§ Calculate the maximum score for the full examination. This may be corrected. However, tests are often graded on a scale of 100 percent.

§ Simplify every option as much as possible. This is the case when questions are of equal weight and responses are also graded equally. However, if the questions are not weighted equally, the marks per question will vary. This should be specified on the exam question paper for the student's benefit.

§ When assigning points, each question's format should be taken into consideration. If a question comprises component components, the weight of each component will determine how many points are awarded.

§ Aim to make your grading rubric accessible to non-specialists in the subject

§ Aim to design your grading plan such that anyone may grade supplied answers and reach a consensus within two marks.

§ Allow for consequential marks; for instance, if a candidate makes an early error but subsequently recovers and proceeds correctly, allow for some marks to be awarded for guaranteeing right steps even if the final answer is substantially incorrect.

§ Pilot your marking system by displaying it to others for their feedback and revisions, which should be incorporated into the final design.

§ Consider what others have done in the past so that you may write your own marking scheme with experience.

§ Learn from your past errors. No marking scheme is flawless; as soon as you begin applying it to scripts, you will begin to modify it, make a note of any challenges you have sticking to it, and record them for future time.

Student Exercise

1. What does test administration entail?
2. Outlines test administration techniques
3. Explain system marking
4. enumerate the main components of an effective marking method in mathematics instruction.
5. List five arguments for using a marking scheme

22.5 Points on the Essay Exam

Generally, essay questions in mathematics are regarded as the classic style of inquiry, which may need brief or extensive responses. A question of the essay kind demands

students to articulate theorems or formulas, apply them, and arrive at an accurate solution, as the work may need. There are two sorts of grading methods that may be applied to essay tests given to students: holistic and analytical.

i. Holistic Scoring System: a method introduced in the 1960s and adopted by the people in the mid-1970s. A holistic scoring system is one that offers an examinee with a single score based on the overall quality of their work (or performance). It provides pupils with a single overall grade for the whole paper. This technique evaluates and integrates a single score or grade issued by the examiner. The approach merely verifies if the candidate is actually answering the questions and completing the task criteria.

ii. The Analytic Scoring Method is a technique for evaluating student work that requires providing a different score to each task dimension. It is most frequently employed when evaluating how well pupils do on individual aspects of a full product or performance. This technique examines several aspects of problem-solving, including as formula or theorem formulation, value sorting, application, and calculation. In addition, the technique provides a more detailed description of the examinee's performance than a holistic score. When thorough feedback is necessary, analytics-based scoring is preferred over holistic scoring. The approach provides detailed feedback that enables examiners (teachers) to determine which aspects of an essay examination kids excel and struggle with. This would benefit teachers in carrying out subsequent exercises.

22.6 Objective Test Points

Depending on the type of objective exam, many methods can be used to score objective tests. Manual scoring, scoring using a stencil, and machine scoring are all methods for determining the test score.

i. Manual Scoring

The columns of responses on each examinee's test are compared to the columns of answers on the master copy as part of this scoring procedure.

ii. Stencil Scoring

By making holes where the correct answers should be on a response sheet that is still blank, a score stencil is produced using this method. Each answer sheet is covered with the stencil to score, and the response checks that fall into the holes are counted. Each exam paper is scanned after this scoring process to make sure there are no multiple-answer errors.

iii. Machine Scoring

Using a machine with a computer and other suitable scoring equipment to score the test elements for big classes, particularly mathematics classes, is another approach for grading objective tests.

22.7 Conversion of Scores to Grades

Typically, a grade is determined by the amount of points earned. Grading is the awarding of grades to pupils' scores. Grading is the process of comparing measurement findings to a'score of reference used' to determine the value of those data. Strong evidence demonstrates that despite the fact that schools strive to implement honest and fair grading procedures, their actual practices differ greatly from institution to institution. Grading is essentially an exercise in professional judgment by educators especially mathematical instructors. It entails gathering evidence on students' success or performance throughout a given time period, such as a term, semester, year, or the full program. This procedure translates many sorts of descriptive information and performance measurements into grades or marks that describe pupils' achievements. Although some mathematics instructors differentiate between grades and marks, the majority consider them to be interchangeable. Both terms refer to a system of symbols, phrases, or numbers used to denote varying degrees of success or performance. They might be letter grades such as A, B, C, D, E, and F; symbols such as CO, NA, +A, B+, and C+; phrases such as satisfied, pass, merit, excellent, good, and fail; or numbers such as 4,3,2 and 1. Below are examples of grading practices at various colleges.

Raw Score	**Letter Grade**
70 and above	A
60---69	B
50---59	C
49---40	D
39 and below	E

Some institutions used grades that correspond to different scores as described below:

Score	**Grade**	**Description**
70 and above	A	Excellent
60 - 69	B	Good
50 - 59	C	Fair
40 - 49	D	Pass
39 and below	E	Fail

This description assists in interpreting student performance in the prescribed examination. Other form of grading systems is used as also shown below:

Score	**Grade**	**Description**

70 and above	A	Excellent
60 - 69	B	Good
50 - 59	C	Merit
45 - 49	D	Pass
40 - 44	E	Low Pass
39 and below	F	Fail

Some schools use this grade system to inform the performance of students. These grades tend to simplify the reporting of performance although they do not reveal the strengths and weaknesses of the student.

Score	Grade	Description	Grade Point
70 - 100	A	Distinction	5
60 - 69	B	Credit	4
50 - 59	C	Merit	3
45 - 49	D	Pass	2
40 - 44	E	Low Pass	1
0 - 39	F	Fail	0

These grade systems present an easy and simple interpretation of students' performance in various examinations as case may be.

22.8 Item Analysis

Item analysis, also known as internal consistency, is the evaluation of test quality by assessing student replies to specific examination questions. The purpose is to evaluate the quality of the objects and the inspection as a whole. The objective of test item analysis is to determine and enhance the test's validity and reliability. It focuses primarily on four factors: test score reliability, item complexity, item discrimination, and distracting information.

22.8.1 Test Rating Reliability

Sometimes, reliability refers to the constancy of a measurement. It is the degree to which a test assesses what it measures consistently, or the amount to which it can yield the same scores on several times. A test is considered dependable if it consistently yields the same result. Typically, the consistency of a measurement is stated numerically as the reliability coefficient. There are four ways to determine the degree of test score reliability: Test-re-test, Split-half, Parallel Test, and Kuder Richardson's Formular k20.

22.8.2 Index of Difficulty

Sometimes, difficulty index is referred to as the ease %. It is the proportion of pupils in the higher and lower groups who properly answered a question. In other words, it is the proportion of pupils who properly answered a certain exam question. If the difficulty

index is high, the question is simple; conversely, the lower the difficulty index, the more challenging the question item.

Index of adversity Formula (p) = n/N or U+L/N

n = Number of students who answered a test question correctly OR Number of students picking the right answer in the upper and lower groups

N = Total Number of Students Who Responded to the Exam Question

Level of Difficulty

Index Range	**Difficulty Level**
0.00 – 0.20	Very Difficulty
0.21 – 0.40	Difficulty
0.41 – 0.60	Average/Moderately
0.61 – 0.80	Easy
0.81 – 1.00	Very Easy

Let look at an example, in computing an item difficulty if in a class of 56 if 15 students answered correctly to a test item: 15/56 = 0. 3

While if the spitted class into upper and lower half members using other approach is shown in Table below:

Table 24.1 Responses for 46 students in a class

Responses	A	B	C	D	E
Upper Half	8	9	5	2	4
Lower Half	0	6	4	10	8

If B* stands as the correct answer in the given item, then

The difficulty index (p) = U+ L / N = 9+6/56 = 15/56 =0.3. Difficulty index is used to determine which item is too easy or too difficult. If an item is too easy or too difficult, it should be discarded or modified if it is to be used in further. The ideal difficulty index is average index, which range from 0.41 to 0.60.

22.8.3 Discrimination Index

This index measures the item's ability to distinguish between students who performed well on the exam as a whole and those who did not. The index helps determine how well the test item distinguishes between smart and average students in the same class. Positive discrimination, negative discrimination, and zero discrimination are the three forms of discrimination.

Positive discrimination occurs when more pupils in the upper group correctly answered the question than those in the lower group.

When more pupils in the lower group correctly answered the question than those in the higher group, this is known as negative discrimination.

Zero Discrimination occurs when there are equal numbers of students in the upper and lower groups who properly respond to the test item are equal.

Level of Discrimination

Index Range	Discrimination Level
0.19 and below	Poor item, should be removed or modified if needed
0.20 – 0.29	Marginal item, needs some revision
0.30 – 0.39	reasonably good item but possibly for improvement
0.40 and above	Very good item

Discrimination Index formula (D) = $C_{UG} - C_{LG}/N$

D= Discrimination index value

C_{UG} = number of students selecting the correct answer in the upper group

C_{LG} = number of students selecting the correct answer in the lower group

N = the total number of students that are involved in test

Using the above example to compute the discrimination index as:

(D) = $C_{UG} - C_{LG}/N$ = 9-6/56 = 3/56 =0.05

A test with many poor questions will give a false impression of the situation. Usually, a discrimination index of 0.4 and above is acceptable but item which discrimination negatively is bad.

22.8.4 Distractor Index

This is the power of distraction or disruption of the students from guessing the correct. Distractor index is the test used for the incorrect alternatives in the multiple-choice type of test, aimed to measure the degree of closeness of the wrong options and the reliability of each questions answer choices. The index normally reads negative because it is indices that differentiate between the performance of brilliant and dull students. A positive distractor index is accounted as poor distractor.

To compute the distractor, index the formula is given as:

(dt) = $B_{UG} - B_{UL}/n$

dt = Distractor index

BU = number of the upper group who choose the wrong options or fail the test item.

BL = number of the lower group who choose the wrong options or fail the test item.

N = total number of students in each group

Using the table below to compute the distractor index

Table Tallying Responses for Mathematics Test

Responses	A	B	C	D	E
Upper	2	5	1	11	1
Lower	3	6	1	8	2

If alternative D is the correct answer then we have: dt) = $B_{UG} - B_{UL\backslash}$ n

= 9 -12/20 = -3/20 = -0.15

Student Activity

1. Explain the following method of scoring i. holistic ii. Analytic
2. What is item analysis
3. Enumerate three indices involved in test analysis
4. What is positive discriminating index
5. Compute difficulty index. Discrimination index and distractor index for the responses given that B is the correct answer

Alternatives	A	B	C	D	E
Upper	2	10	0	0	3
Lower	4	5	4	0	2

6. Ten students have taken a mathematics test of ten objective questions as shown in the table below:

Table 2: Maths Test Scores

Question	1	2	3	4	5	6	7	8	9	10
Upper	4	5	4	4	5	4	4	4	2	4
Lower	2	5	4	1	3	2	1	2	2	2

a. Which question was the easiest?

b. Which question was the most difficult?

c. Which item has the poorest discrimination?

d. Which questions would you eliminate first (if any) why? Calculate the Difficulty Index(di) and Discrimination index

7. Enumerate and explain the methods of scoring objective test.

22.9 Summary

The major points raised in this chapter include the following:

§ Test administration which means the range of activities developed for an examination in order to help reduce measurement errors and increase fair, validity and reliability. This covered from activities from beginning, during and after the test.

§ Marking scheme as an outline of the expected answers together with the mark allotted to each question.

§ Two scoring methods in essay test and these are holistic and analytic

§ Manual, Stencil and Machine are scoring methods in scoring an objective test

§ Grades tend to simplify the reporting of performance although they do not reveal the strengths and weakness of the student. Grading is the process of comparing the measurement results with as 'score of reference used' that results form of value.

§ Item analysis is an act of analyzing student responses to individual examination questions with the intention of evaluating examination quality.

§ Item analysis focused on four major items: score reliability, item difficulty, discrimination and distracter.

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CHAPTER TWENTY-THREE

CONTINUOUS ASSESSMENT

23.1 Objectives

By the end of this chapter, you should be able to:

i. Explain why C.A was introduced into our school system
ii. Define the concept of C.A
iii. Characteristics of C.A
iv. Advantages of C.A

23.2 Introduction

C.A is a veritable tool because of the decisions you will make about student learning and your teaching. Every day in your school time, you will be called upon to make decisions before, during and after teaching. All of your assessment decisions taken as a whole will direct and alter students' learning outcomes. Teaching and learning can be enhancing through assessment.

23.3 Reasons for C.A in our School System

The Federal Government of Nigeria has been made aware of the deficiencies of the previous one-time summative evaluation of student performance, which has led to the implementation of ongoing assessments in our institutions. The method of evaluating students' ability or performance will replace the previous one-shot provided at the conclusion of each term or school year for the purpose of class advancement. Similarly, at the conclusion of a program or course of study, an external agency (such as WAEC and NTI) administers a test for certification reasons.

In the previous method, only intellectual capacity (cognitive) is assessed, ignoring other domains, such as the emotional domain, which deals with the learners' interest, attitude, motivation, emotion, and worry. Also, the previous method seldom considered the psychomotor domain in its whole.

In the old one-time summative review, there was no plan for assessment. In several schools, evaluations were conducted hastily. Learners view the final examination as a life-or-death situation because the previous method provided only a single evaluation and was extremely stressful. This results in examination irregularities and a wide range of standards.

In the previous system, there was no method to address obstacles such as illness or other obstacles that prevented students from sitting the final examination. This sometimes left students at a disadvantage, since they were had to redo the full year or course. External examination bodies perform the final examination for certification of

students, therefore teachers are not engaged in its administration. You will support the establishment and implementation of continuous assessment in schools based on the aforementioned arguments.

23.4 Definition of Continuous Evaluation

Continuous assessment (CA) has been characterized in a variety of ways by various academics. According to some experts, CA merely refers to periodically assessing or weighing student performance in order to identify the progress achieved in teaching-learning activities. It is a sort of educational testing that examines the students' development or performance during the course of their schooling. CA requires the continual and systematic maintenance of accurate records on each student. It offers feedback to mathematics teachers about their performance and measures pupils' learning progress.

Importantly, since instructors construct, administer, and evaluate continuous assessments on topics of class teaching that are relevant to them, there is a high possibility that they will use the results for their personal benefit. With these definitions, CA provides an overall assessment of the educational system. In other words, decisions are made based on the facts acquired and examined throughout the evaluation. Therefore, judgments are more scientific and less speculative.

23.5 Qualities of Continuous Evaluation

There are defining characteristics of CA. These key characteristics include exhaustive, cumulative, methodical, learner-centered, guidance-oriented, and predictive.

i. Systematic: CA is considered to be systematic since it needs an operating plan that specifies what types of assessments will be performed and when they will be utilized.

ii. Comprehensive: The CA is considered comprehensive since it assesses not only cognitive ability, but also emotional and psychomotor domains. Thus, there is a comprehensive evaluation of the student. This also raises the responsibility of the instructors who look at the full three domains while assessing.

iii. Cumulative: CA is considered to be cumulative since records are added as the work of the term proceeds. The majority of documents maintained during and during the time were not disregarded. The old, one-shot method, however, tended to focus solely on the evaluation at the end of the term or even the year, ignoring the maintained data.

iv. Oriented toward guidance: The record obtained via CA is utilized to guide an individual student's profession choice depending on his or her aptitude.

v. Learner-Centered: CA focuses teacher and student attention largely on monitoring and improving instruction. It gives instructors and students with

information to assist them in making modifications to enhance learning.

vi. Prognostic: CA is prognostic in the sense that information gathered during diagnosis can be utilized to forecast how well a student will do in the future on comparable tasks or on problems that are entirely different.

23.6 Benefits of Continuous Evaluation

Now, let's review the benefits of CA. This will allow us to comprehend the implementation of CA in schools.

i. CA takes into consideration the actions and performance of pupils during their whole academic career. Therefore, it is a better indicator of pupils' overall ability.

ii. It gives the possibility for practical teachers to be directly involved in the final certification of pupils. In order to maintain the reliability and validity of student certification, teachers are urged to be creative and devoted in the classroom, as well as to apply CA with the utmost honesty.

iii. CA generates complete data for rectifying students' learning shortcomings and providing suitable advice and counseling services.

iv. It supports the instructor in enhancing the instructional process. For instance, the feedback compels the instructor to periodically evaluate the educational approaches in order to enhance the teaching procedure.

v. It helps pupils to cultivate effective study habits.

vi. CA fosters entire growth of pupils since all parts of learning experiences, behaviors, skills and all other activities are being examined.

23.7 Issues with Continuous Evaluation

As part of the 6-3-3-4 education system in Nigeria, continuous evaluation was implemented. Due to the fact that it is a novel innovation, complications are inevitable. Consequently, let's consider a few issues with the implementation of continuous evaluation in our institutions.

i. Creation and Application of Standardized Instruments

Comparisons of results among Nigerian schools are hampered by disparities in the quality of evaluation instruments and methodologies employed in various schools, as well as scoring procedures. Again, instructors in the school system frequently lack enough training in the preparation and implementation of exams and other instruments, resulting in varying institutional practices. There is a challenge in implementing and developing a standard tool with unqualified teachers. Therefore, they operate in the shadows and identify all of their actions as CA.

ii. Improper Record-Keeping

Teachers in the school system are frequently too occupied with conventional and extracurricular activities to dedicate sufficient time to correctly record student tests. They appear unmotivated to maintain such documents and the amount of effort required in CA, which exceeds their remuneration.

iii. Storage Facilities

We need room for documents related with CA for safety keeping but regrettably such space does not available in many schools. As a matter of fact, most elementary and secondary schools have such difficulty, and the records are not effectively guarded.

iv. Large Class Size

In many of our institutions in Nigeria, the practice of CA is hampered by the large number of pupils per classroom. The teacher-student ratios are not adhered to, preventing a comprehensive assessment of each kid in the class across all behavior categories.

v. Computational Ability

Inadequate computational ability is another issue with the application of CA. The present practice of CA necessitates the translation of scores into a standard form; yet, a significant proportion of teachers are unable of doing this work. Therefore, individuals rely on the approach with which they are familiar.

vi. Lack of Authenticity

Teachers are deemed insincere if they assign grades to pupils based on favor or in exchange for a reward. All of these actions demonstrate a lack of dedication to the adoption of CA practice.

Student Activity

1. List four ways assessment was done in the old system
2. Explain what is meant by one-shot assessment
3. Mention two disadvantages of old system assessment
4. List four advantages of continuous assessment
5. Give two definitions of continuous assessment
6. Give three reasons for introduction of continuous assessment in school system
7. What do we mean by bringing records of assessment forward?

8. Enumerate five characteristics of continuous assessment

9. There is the problem of unqualified teachers to implement the continuous assessment. Discuss

10. List five problems of continuous assessment implementation in Nigeria.

23.8 Summary

The following are points to note in this chapter:

§ Continuous assessment was introduced in the school system to replace the old one-shot system of assessment. It based on three educational domains.

§ Continuous assessment is a commutative about performance of each student in the class. It makes the student to work hard not just toward the end of semester or term.

§ Continuous assessment makes use of different instruments, such as test, observation, socio-metric, project and questionnaire.

§ Continuous assessment has major characteristics which are it is comprehensive, cumulative, systematic, guidance oriented, learner centre and prognostic

§ Continuous assessment has been surrounded with several problems that affect its smooth implementation, such development and use of standard instrument, large class size, storage facilities, lack of sincerity, inadequate computation skill and so on.

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